DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

COAST EROSION AND THE DEVELOPMENT OF BEACH PROFILES

TECHNICAL MEMORANDUM NO. 44

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JUNE 1954

FOREWORD

Important factors to be determined in coastal engineering studies are the equilibrium beach profile, the rate of shore line recession due to wave action, and the resultant position of the shore line at some time in the future, including of course, the effects of shore structures on these determinations. A great deal of work, both in the laboratory and field, has been done on these subjects by workers abroad, little of which is easily available in English to engineers in the United States. The report which follows is published to disseminate information on some such work in Denmark, and on some of the ways in which shore line problems are attacked by engineers abroad.

This report was prepared by Mr. Per Bruun, a Danish coastal engineer with the Technical University of Denmark, during a portion of the time in 1952 and 1953 that he spent in the United States.

Views and conclusions stated in the report are not necessarily those of the Beach Erosion Board.

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TABLE OF CONTENTS

Part I - The Danish North Sea Coast

Page

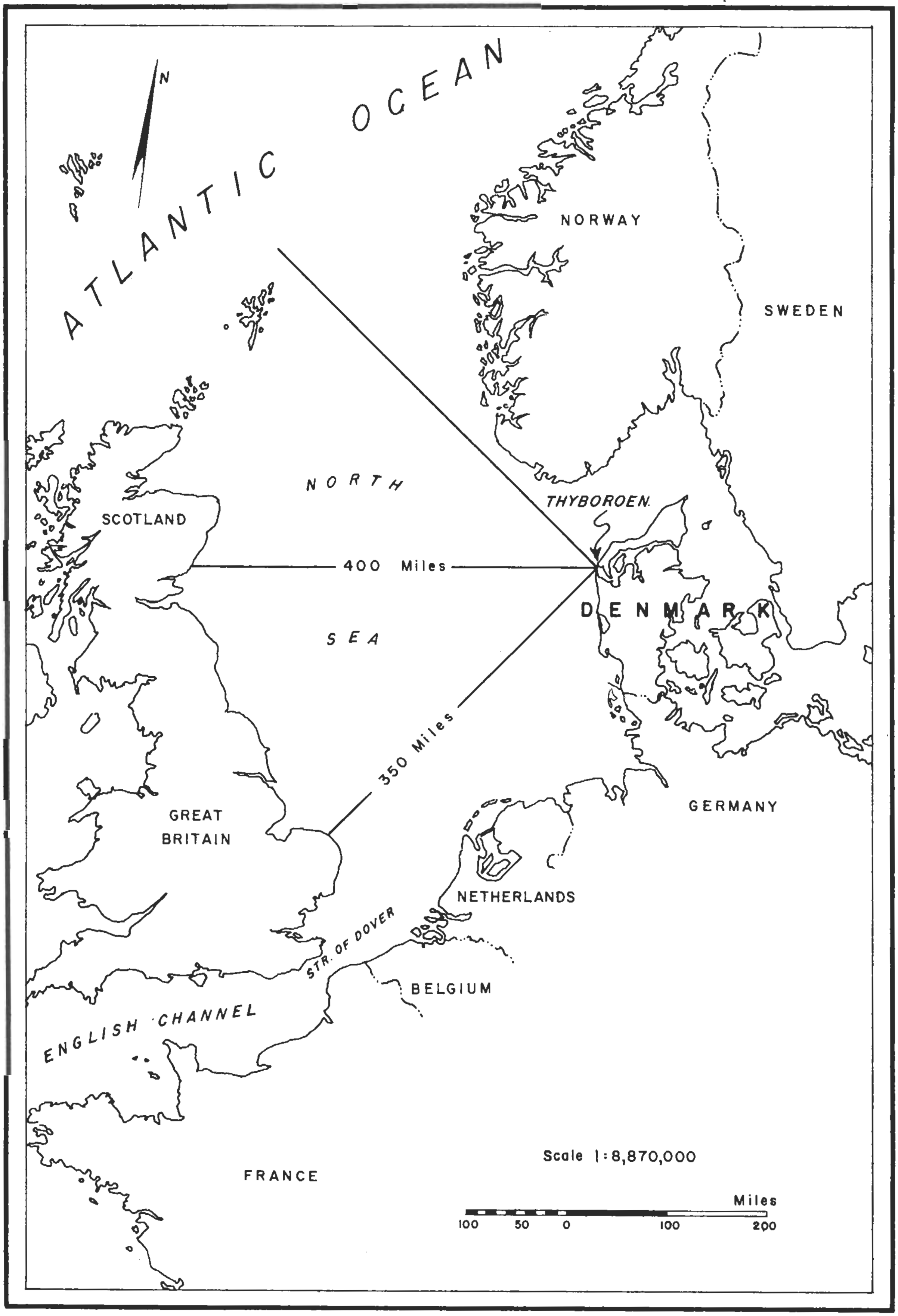

FIGURE I REGIONAL MAP

COAST EROSION AND THE DEVELOPMENT OF BEACH PROFILES

by

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ABSTRACT

Part I of this paper consists of a study of the Danish North Sea coast. The following factors are included:

a. Coastline development;

b. Development of beach profiles, including comparison for different wind/wave conditions;

c. Coast erosion and quantity of littoral drift;

d. Forecasting the future development of shore line and beach profiles on the basis of these investigations.

Depth soundings since 1874 on the Lime Inlet Barriers on the Danish North Sea Coast are used and treated statistically. In this way the development of the Lime Inlet Barriers and adjacent coasts is explained.

Part II consists of a study of the Mission Bay, California area. It includes study of:

a. Development of beach profiles, including comparison for different wave conditions;

b. Seasonal fluctuations of beach profiles;

c. Comparison of Danish and California data.

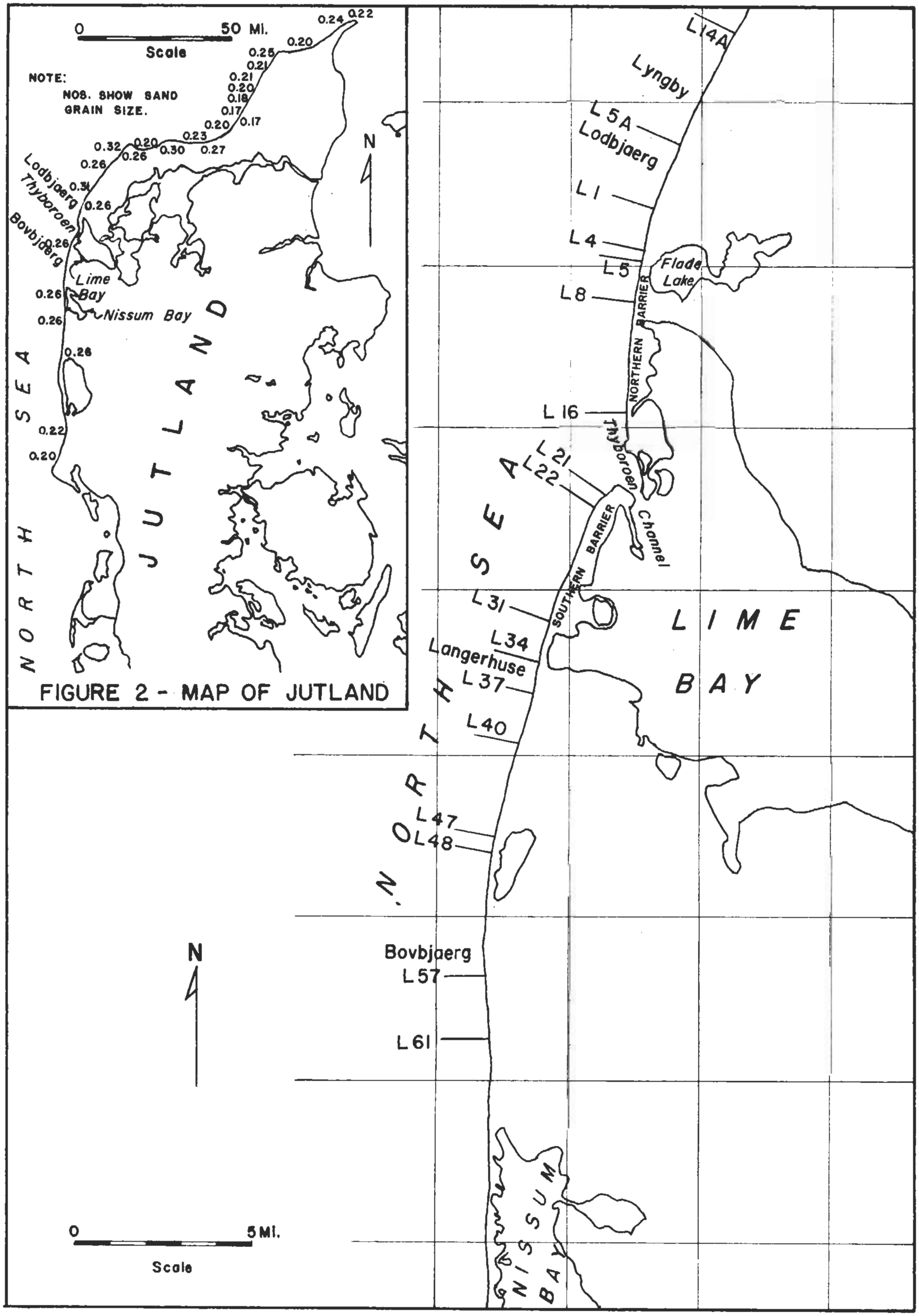


FIGURE 2 - MAP OF JUTLAND

FIGURE 3 - VICINITY MAP

PART I - THE DANISH NORTH SEA COAST

1. GEOGRAPHY AND GEOLOGY

1.1 GEOGRAPHY. A regional map of the North Sea is shown in Figure 1. On this map, lines have been drawn from Thyboroen in northwest, west and southwest directions showing the fetch distances between Thyboroen and the English coast. It may readily be seen that there is free communication to the Atlantic Ocean on the northwest.

1.2 GEOLOGY. The peninsula containing Jutland (the mainland portion of Denmark) is shown to a larger scale in Figure 2. It is composed mainly of Pleistocene deposits, chiefly glacial drift (boulder clay, meltwater sand, and clay originating from three separate glaciations). In the northernmost region post-glacial "marine-foreland" accumulations also occur. The west coast is composed of relatively weak materials (mainly sand), and has been shaped by the action of the waves and currents into a smooth and plain shore line. The Lodbjaerg headland north of Lime Inlet (Thyboroen Channel) is a nodal area for the nearshore littoral drift along this coast. Figure 2 also shows the grain size variation along the west coast as determined for Bascom's "reference point" (23)*. The grain size decreases in both directions from the nodal area, except for the headlands.

1.21 The Lime Inlet Barriers at Thyboroen and the adjacent coast are shown to a larger scale in Figure 3. These Barriers are composed of sand and reach a height of about 5 feet. The material below -20 feet is a very soft Litorina clay. Sizeable moraines are found on the two headlands, Lodbjaerg (North of the Barriers) and Bovbjaerg (South of the Barriers).

2. HISTORY

2.1 ANCIENT HISTORY. Historical accounts suggest the existence in the 11th Century of a good navigable connection with the sea of Lime Inlet, through which the Viking raiders passed on their way to England. Indeed, there is perhaps some slight connection between the cessation of these raids and the shoaling of the channel.

2.11 The appearance of the landscape behind the dunes as pointed out by Tessen (10) makes it probable that a North-South oriented channel may once have existed in this area. This would support the theory of a southward drift of material on the unbroken Barrier.

2.2 RECENT HISTORY. The past 200 years can be separated into two distinct periods; one in which no permanent channel existed, and another occurring later in which such a channel did exist.

* Numbers in parentheses refer to bibliography on pages 78-79.

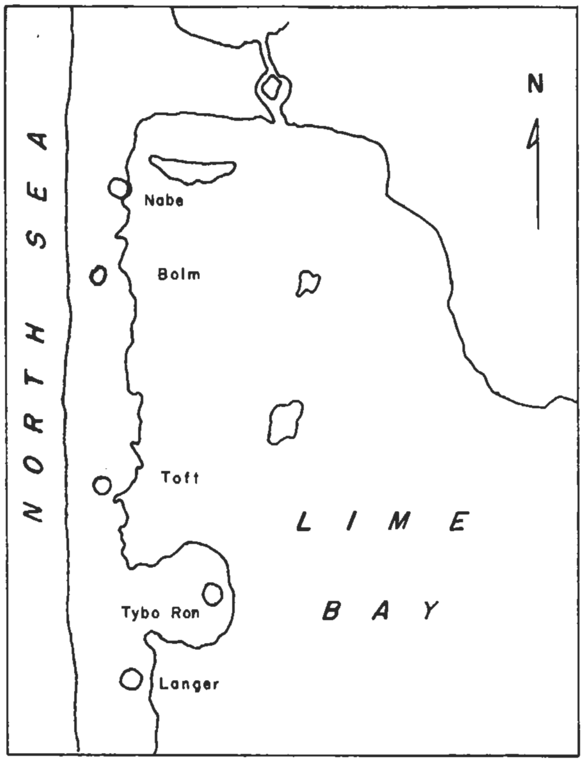


FIGURE 4 – 1695

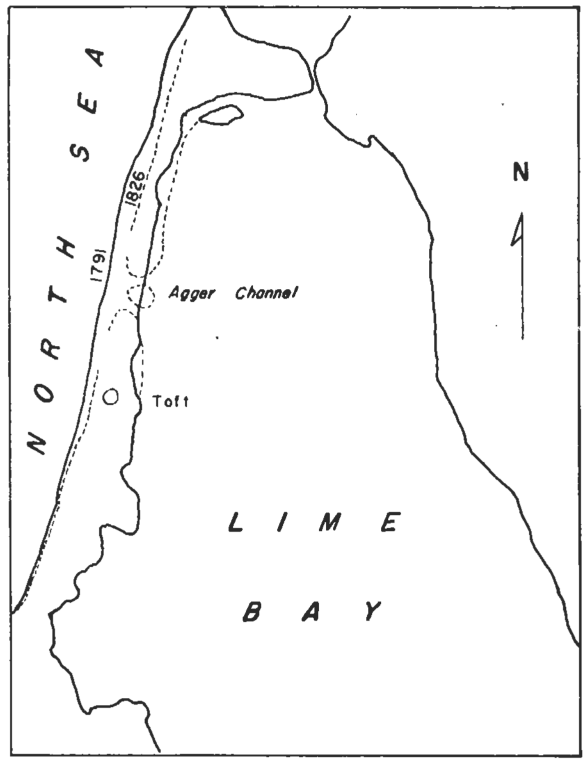


FIGURE 5 – 1791 & 1826

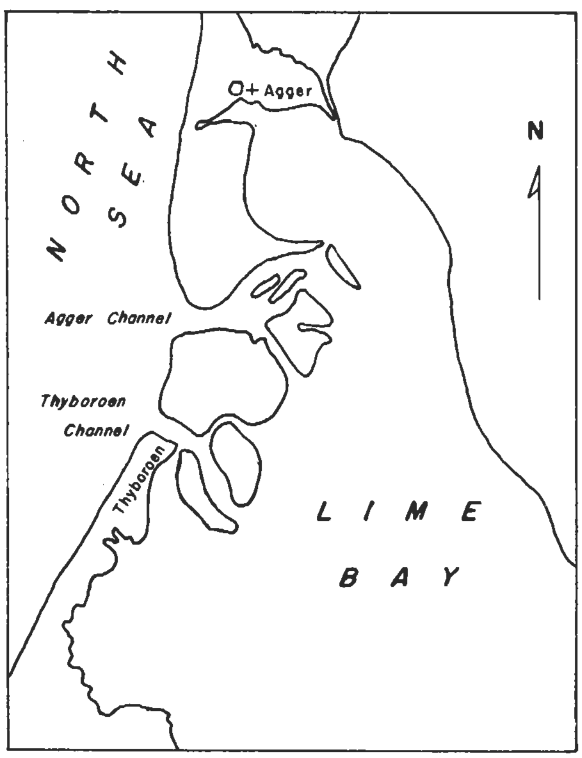


FIGURE 6 – 1867

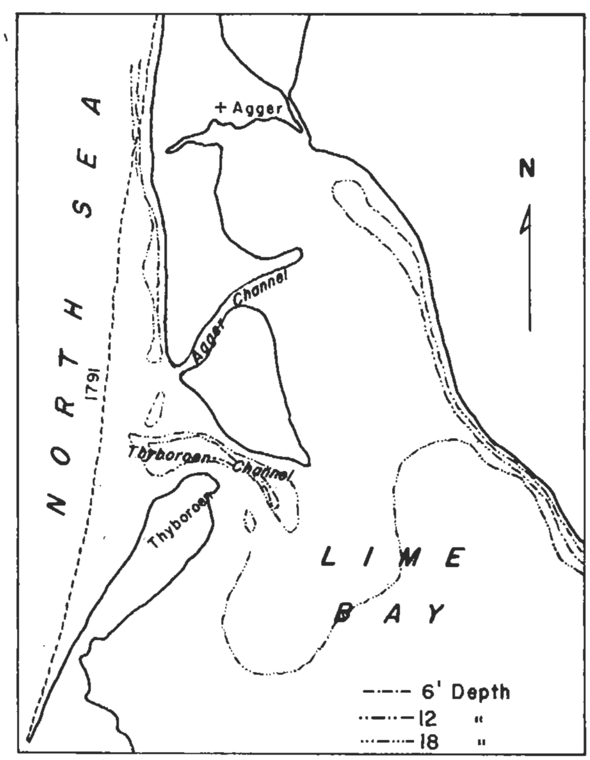


FIGURE 7 – 1874

0 1/4 1/2 Mile

HISTORICAL SURVEYS

2.21 The period without any permanent channel. The first relatively detailed survey obtained here was that of Jens Sorensen in 1695, (Figure 4). It reveals that at that time the Barrier was unbroken, although historical accounts indicate frequent wash-outs of the dunes. Figure 5 shows the survey data obtained by the Academy of Science in 1791. Although it is difficult to make a comparison between these two surveys, it appears as though the coast receded an average of about 10 feet a year during that period. The surveys locate several villages on the Barrier all of which were small and poverty stricken. Frequent inundation of the Barrier forced the abandonment of at least two of these, first Bolm, and later, in 1775, Nabe.

A comparison between measurements made in 1791 and in 1938 shows that the shore line receded at average annual rates of about 9 feet at Lodbjaerg and 7 feet at Bovbjaerg.

2.22 The period with a permanent channel. The unbroken Barrier was gradually narrowed by erosion, while the planting of the dunes with marram grass retarded the landward sand drift, resulting in a gradual destruction of the dunes by the sea also.

A very severe storm on the night of February 3, 1825 washed out most of the remaining dunes, and necessitated the abandonment of the village of Toft. Another intense storm on the night of November 27, 1825 breached the Agger Channel. Its enlargement continued until 1849 when it began shoaling. A heavy storm in 1862 formed the Thyboroen channel and after this the Agger Channel gradually shoaled. Figures 6 and 7 show the two channels in 1867 and 1874 respectively. In 1875 the Agger channel closed completely by sanding up. The remains of the channel may be seen in Figure 3.

As soon as a new channel is formed, sand from both sides of the channel moves into the inlet where it is deposited in large shoals; a typical situation of this kind, existing in 1941, is shown in Figure 8. Also included in this Figure is the shore line of the unbroken Barrier in 1791. That part of the channel inside the Barriers migrates slowly westward, and is very difficult to maintain because of shoaling (Figure 8). Table 1 shows the volume of sand deposited in these shoals.

TABLE 1 - SAND DEPOSITED IN THE SHOALS

Years	Volume (Million cubic yards)	Rate (Million cu.yds./yr.)
1912-1917	1.7	0.3
1917-1924	11.0	1.6
1924-1930	4.8	0.8
1930-1937	5.8	0.8
1937-1943	9.2	1.5
1912-1943	32.5	1.0

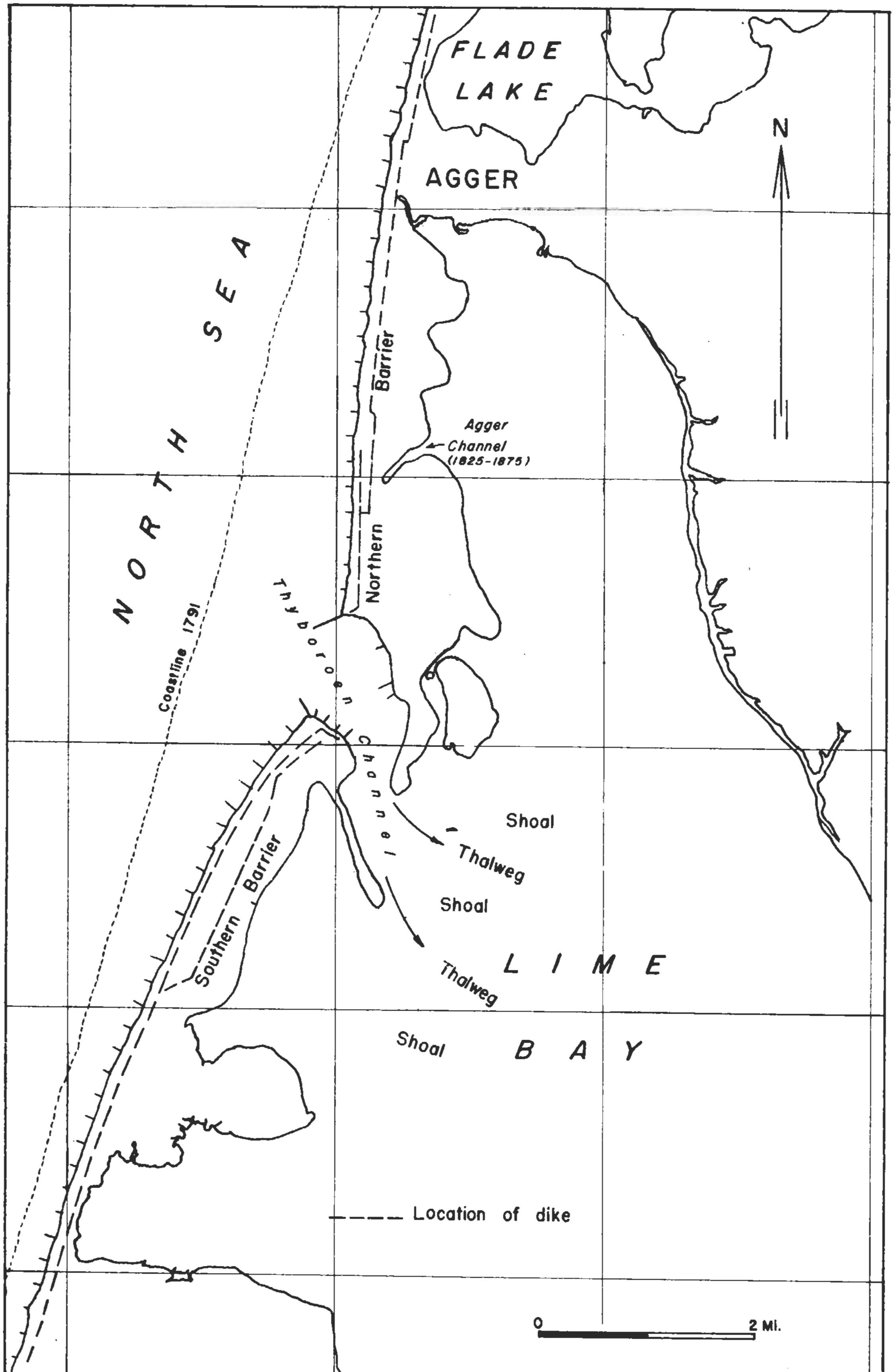


FIGURE 8- LIME BAY AREA (1941 SURVEY)

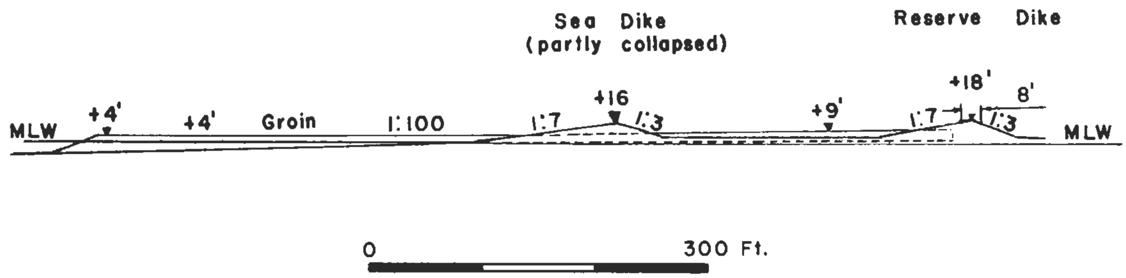
It is difficult to establish the location of the nodal points for the nearshore littoral drift in this area, but deposits against the groins on the Barriers seem to show that the nodal point on the Northern Barrier is situated not very far from Lodbjaerg, near L 1 in Figure 3, and that the nodal point on the Southern Barrier is situated about 6 miles south of Thyboroen, between L 31 and L 37. Furthermore, it is probable that a northward littoral drift, due primarily to currents other than those generated by waves, occurs in the deeper water seaward of the Barriers.

2.3 COASTAL PROTECTION. During the period 1875-1909, 30 groins were built on the Southern and 25 on the Northern Barrier, in addition to the large dikes and groins constructed along the channel (Figure 8). Moreover 23 additional groins were constructed at Bovbjaerg in the period 1875-1937. All these works are financed, built and maintained by the Danish State, as is the harbor development at Thyboroen since 1915. The dikes are designed to protect the barriers from wash-outs, and the groins to check losses by longshore drift. The spacing of the groins is about 1,250 feet and their length varies from 700 to 1,300 feet, generally increasing towards the channel. The end groins on both sides of the channel are now built as jetties 1,400 and 3,000 feet long respectively (Figure 8). These groins are no longer maintained to their full length, the continued erosion and resultant high cost of maintenance having forced the abandonment of 200 to 400 feet of the seaward ends.

The design and construction of groins and dikes have been gradually developed and improved over the years. Figure 9 shows a cross-section of the coastal protection on the Southern Barrier, which, like the Northern Barrier, has both a "sea dike" and a "reserve dike". Groins (Figure 10) are generally built of a core of piled-up concrete blocks -- the "crown" -- with additional concrete blocks or rubble heaped along the sides (4,7). The weight of a block is usually 4 tons. In exposed places 8-ton blocks are used.

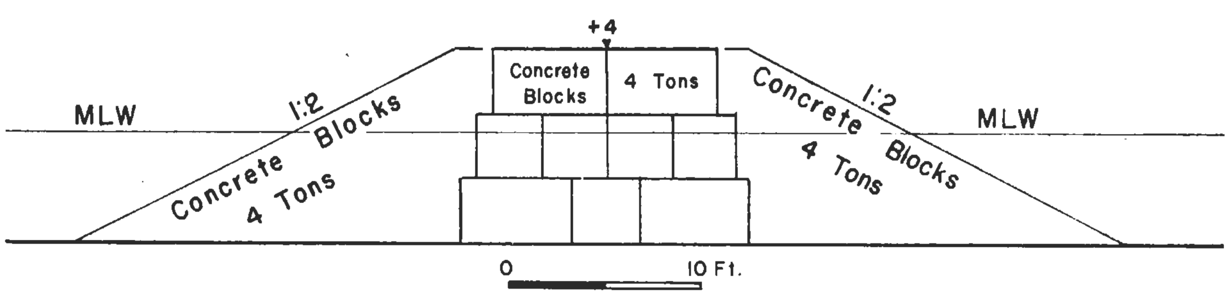
A stepped facing of reinforced concrete (slope 1 on 2) has been placed along about one mile of the dike on the Northern Barrier, north of Agger off Flade Lake and also for a distance of about 1,200 feet at Thyboroen on the spit of the Southern Barrier (Figure 8). In places this design has not proved very durable, partly because the sheet piling was not tight, and partly because the waves swept over the structure and washed away sand behind and under the facing. A flexible type would probably have been better. This construction has been described previously by the author (4, page 165).

The most recent proposed measures against erosion in this area were authorized in the Act of August 1946, which provided for the construction of two large jetties, one on either side of the inlet channel, and a new solid dike about $1\frac{1}{2}$ miles inland. No work has been done on this project, details of which are contained in Appendix A.



SECTION OF SOUTHERN BARRIER

FIGURE 9



GROIN SECTION

FIGURE 10

3. COAST EROSION BETWEEN PROFILES L14A(LYNGBY) and L61(DYBAA)

This particular section of the coast, about 25 miles in length, is shown in Figures 2 and 3. The erosion in this area is primarily caused by wave and current action. The most frequent strong winds observed are those from the west quadrant, a typical gale starting from the southwest with force 4-6 Beaufort and, in the course of about 12 hours, increasing to force 6-9 Beaufort, while veering to the west or the northwest where it may continue for several days. The water level often begins to rise when the barometer falls, even prior to the advent of winds of gale force, because water from the English Channel is forced into the North Sea through the Strait of Dover. When the wind has shifted to the northwest,, the water level falls. The mean tidal range at Thyboroen is about 16 inches, but heavy westerly gales may cause as much as a 5 to 6-foot rise above this. Wave heights, under such conditions, may reach 12 feet, with the average wave length being 300-400 feet. The coastal current in deeper water will usually have a northward direction with a speed of about 1 mile per hour, although speeds of 3 to 4 miles per hour do occasionally occur. In the last twenty years severe storms in 1936 and 1948 resulted in only minor damage to the existing dikes.

3.1 COASTLINE DEVELOPMENT

3.11 Shore line changes between 1791, 1874, and 1938. Figures 11a and 11b respectively show schematically the 1938 shore line in proportion to the rectified shore lines of 1791 and 1874. It may be seen from these figures that the influence area of the channel probably lies between profiles L14A and L48, (Figure 3).

3.12 Shore line changes since 1874. The average annual recession of the shore line on the Barriers, from L1 to L16 on the Northern Barrier to L22 to L37 on the Southern Barrier, is shown in Table 2. The numbers shown represent the averages of 16 different values on the Northern Barrier and 11 on the Southern Barrier, each of these values being obtained from 100 to 150 individual measurements in a limited reach. In this way an exact determination of the position of the shore line is obtained. From Table 2 it may be seen that the rate of recession of the shore line on both Barriers, has decreased, probably because of the construction of the groins, until a minimum value was attained in the period 1927-1934(1938). After this date the rate increased on both barriers which probably means that the beach profiles are now close to their maximum steepness (see section 3.24).

In Table 2 measurements between L16 and L18 have been omitted because the construction of the large jetty on the southern point of the Northern Barrier has materially affected the development of the shore line in this area.

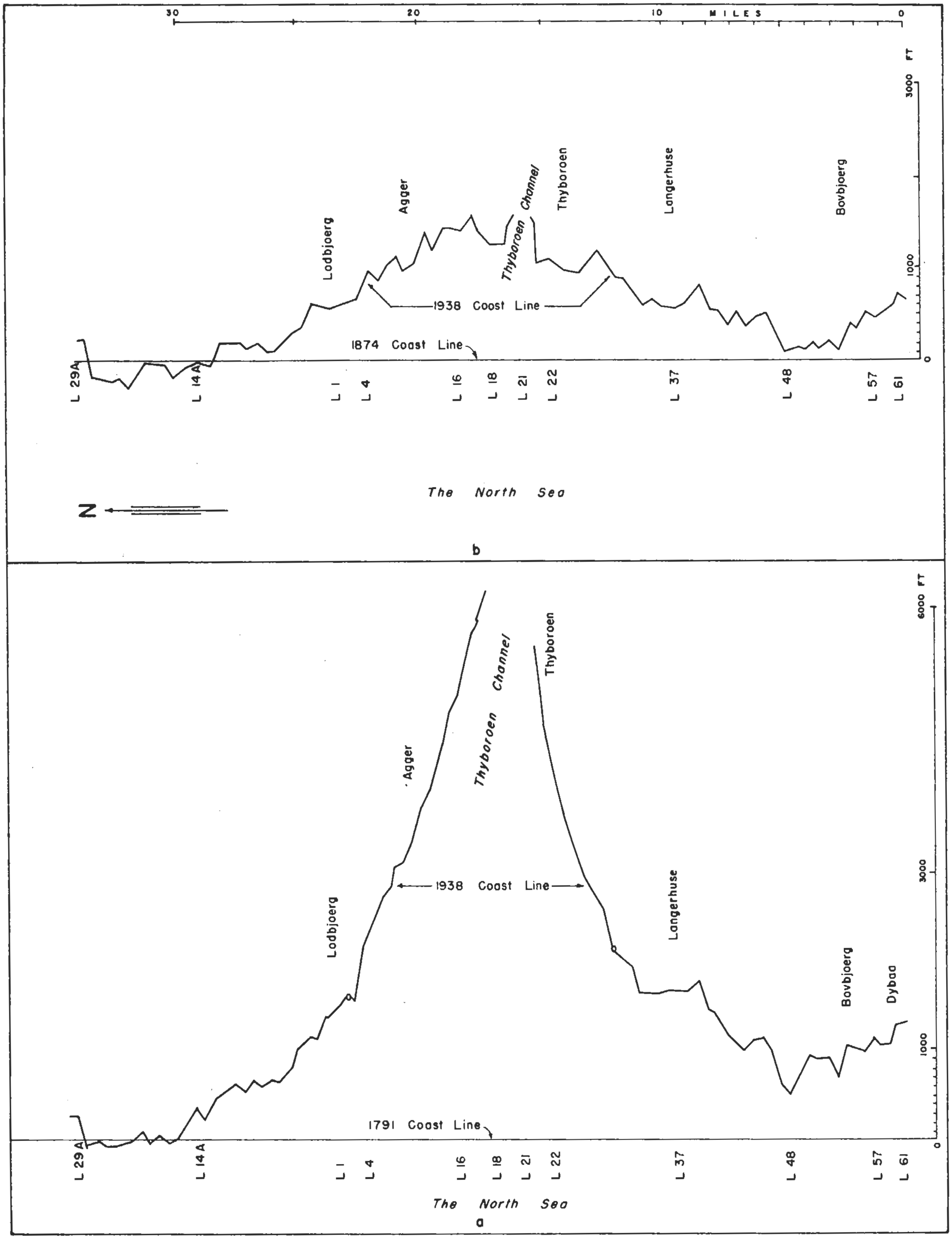


FIGURE 11. RELATION OF 1938 SHORE LINE TO RECTIFIED SHORE LINES OF 1791 AND 1874

TABLE 2 - RATE OF RECESSION OF THE SHORE LINE ON THE LIME INLET BARRIERS IN FEET PER YEAR

Years	Northern Barrier (L1-L16)	Southern Barrier (L22-L37)
1874-1897	29	24
1897-1903	21	7
1903-1909	9.5	12.5
1909-1916	7.5	3.5
1916-1921	10	7.5
1921-1927	9.5	6.5
1927-1934	4.5	1.5
1934-1938	4	6
1938-1942	15.5	5.5
1942-1945	6.5	9.0
1945-1948	12	4.5

Table 3 is a comparison between the recession rates of different portions of the shore line. The location of the profile reaches given in Table 3 may be seen in Figure 3.

TABLE 3 - COMPARISON BETWEEN THE RECESSION RATES OF DIFFERENT PORTIONS OF THE SHORE LINE (FEET PER YEAR)

	SECTIONS								
Years	L 1A to L 14A	L 1A to L 5A	L 1 to L 16 (BARRIERS)	L 22 to L 37 (BARRIERS)	L 38 to L 40	L 38 to L 47	L 48 to L 57	L 58 to L 61	L 38 to L 61
1874-1903	2	5	27.5	21	12	6	5	9	6
1903-1921	5.5	8.5	9	7.5	2	7	2	8	5
1921-1927	8.5	4.5	9.5	6	19	10.5	11	1.5	9
1927-1934	3.5	5.5	4.5	1	9	5.5	1.5	30	8
1934-1938	9	22	4	6	13	13.5	7.5	23	12.5
1938-1948			11.5	6					
1921-1938	6.5		6	4.5	13.5	9.5	6.5	18	9.5
1874-1938	4		16.5	12.5		7			

Shore line conditions north and south of the Barriers are measured only every second year in a single point given by a line of soundings. This determination, therefore, is not as exact as that for the Barriers. The reaches L 1A to L 5A and L 38 to L 40 are considered separately, as then it can be seen (Table 3) that the area of maximum yearly recession has migrated to both sides of the channel. This process seems to have taken place independently of the presence of the groins. Table 3 further shows that the construction of the groins on the Barriers (L4 to L 16 and L 21 to L 37 , see Figure 3) has reduced the annual rates of recession of the shore line from 1874-1903 to 1903-1921 from 27.5 to 9 feet on the Northern Barrier, and from 21 to 7.5 feet on the Southern Barrier.

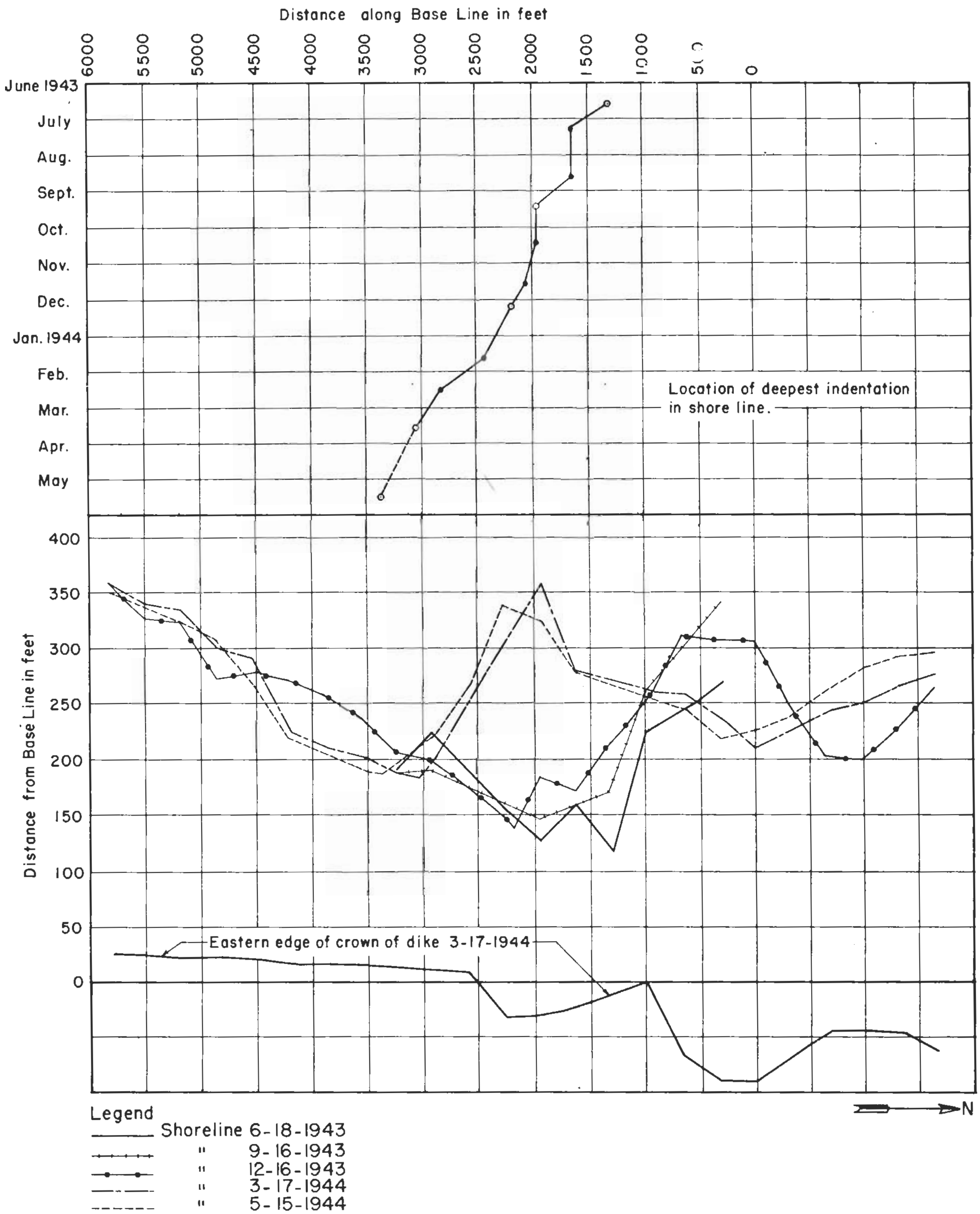


FIGURE 12 - MIGRATING "WAVE" ALONG NISSUM INLET BARRIER

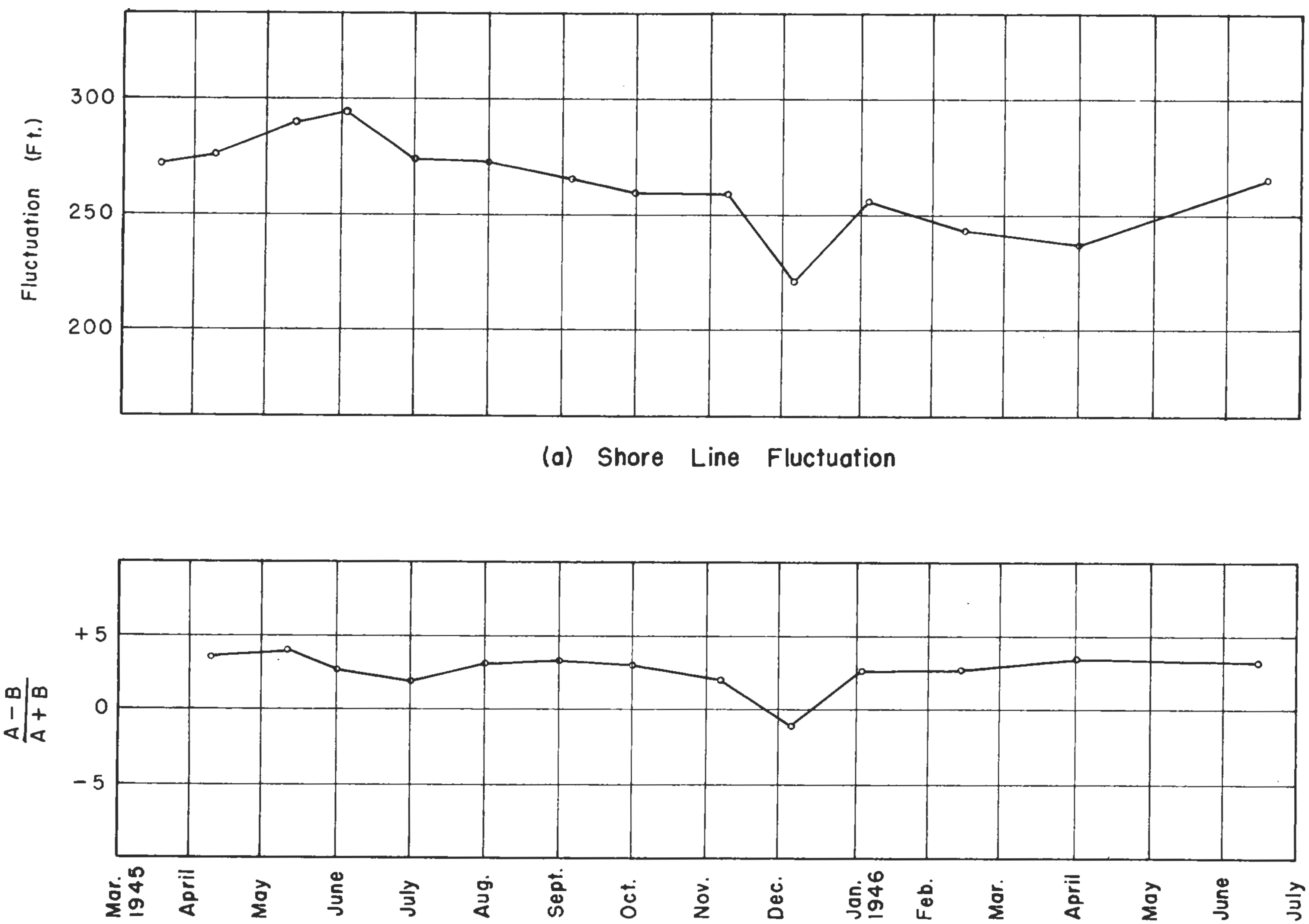
It can also be seen from Table 3 that the minimum rate of recession of the shore line occurred in the period 1927-1934 on protected as well as unprotected reaches, (except for L 58 to L 61, where special conditions prevail; see below). This is probably not a result of good maintenance of the groins in that period, but rather of the incidence of less severe weather conditions, (see section 3.3, Table 15).

The compilations in Tables 2 and 3 are average values derived from much more detailed tables which demonstrate quite markedly the migration of the area of maximum recession to both sides of the channel. Moreover these tables show that this migration started soon after the breach of the Barriers at Thyboroen and hence probably is not connected with the construction of the groins on the Barriers. The increase in recession rate of the shore line on the unprotected coast; L 1A to L 5A and L 38 to L 40, to the maximum observed values in 1934-1938 probably is a result of an extension of the area of channel influence; see the above. The initial heavy erosion of the Barriers near the channel results in an increased curvature of the coastline, and hence in an accelerated rate of erosion farther away from the channel; this process is continually repeated and results in a gradual lengthening of the erosion area (Figure 3).

The increase in recession of the shore line between L 58 and L 61 since 1927 is due primarily to the construction of groins on Bovbjaerg, north of (updrift from) this area. This erosion has been described previously by the author (7, 25).

3.13 Detailed investigations of shore line recession. It is believed that three different types of shore line movement are discernable. These are, in decreasing order of magnitude, migrating waves, seasonal fluctuations, and long period changes caused by erosion.

a. Migrating waves. A shore line is never straight, but rather is composed of many curves or "waves". The length of these waves on the Danish west coast is usually between 300 and 2,000 yards. Periodic measurements show that the wave amplitudes are 60 to 80 yards on the free and unprotected coasts, but are somewhat larger where the coast is protected by groins, possibly because of the storing of material between the groins. Figure 12 shows five different stages in a migrating wave along the shore line of the Nissum Inlet Barrier (see Figure 2). The wave length was about 1,000 yards, the wave amplitude about 60 yards and the trough migrated about 700 yards in one year in the direction of the littoral drift. On that part of the west coast it appears that the speed of the migrating waves will vary between 0 and 1 mile a year. There is undoubtedly a connection between these waves and the rip currents through breaches in the longshore bar. Similar phenomena at Scripps Beach have been described by Shepard and Inman (18).



(a) Shore Line Fluctuation

(b) Wind Condition (See 3.13b for definition)

FIGURE 13-SEASONAL FLUCTUATION OF THE SHORE LINE - NISSUM INLET BARRIER

b. Seasonal fluctuations. Laboratory experiments with beach profiles in conjunction with field studies have shown that waves with a high steepness ratio (e.g. $H_0/L_0 = 0.04$) will tend to erode the beach, whereas waves with a low steepness ratio will tend to build up the beach (1, 3, 12, 20, 25). For this reason, in the usual case, the shore line will prograde in the summer and recede in the winter. Figure 13 shows the seasonal fluctuations of a 2,000-yard section of the Nissum Inlet Barrier shore line. The figure was computed from measurements taken at 19 points 110 yards apart at approximately monthly intervals over a period of more than one year. The seasonal fluctuations here is about 20 yards while on the Southern Barrier it is about 10 yards. The wind velocity was measured three times every 24 hours (Beaufort scale) during this time. If the number of observations of onshore winds with velocity ≤ 4 Beaufort is denoted by A, and the number of observations with velocity ≥ 5 Beaufort is denoted by B, the ratio $(A - B)/(A + B)$ can be computed for each period. This ratio has been plotted in Figure 13b for each survey period. No winds with a velocity > 8 Beaufort were observed during this time and, therefore, the observations in the following considerations are given the same weight which obviously must be incorrect. It is apparent that there is a connection between Figures 13a and 13b.

c. Long period changes caused by erosion. Detailed observations on the Lime Inlet Barriers where the shore line location is determined with great exactitude, show a single beach profile on the Barriers will never recede more than 7 to 8 yards in one year.

Table 4 lists the amounts of the three different types of changes for various reaches between L 14A and Nissum Inlet. The last column of this table shows the maximum average annual movement of the shore line since 1921 for each reach.

TABLE 4 - TYPES AND AMOUNTS OF SHORE LINE CHANGE (YARDS)

Coastal sector	Migrating waves	Seasonal fluctuations	Long period changes due to erosion	Maximum average annual recession since 1921
L 4 - L 14A				3.1
The Northern Barrier	70		7	5.2
The Southern Barrier	100	10	7-8	3.0
L 37 - L 47				4.6
L 48 - L 57				3.8
L 58 - L 61				10.0 (*)
The Nissum Inlet Barrier	60	20		

(*) caused by leeside erosion.

3.14 The development of the coastline investigated by means of mass-curves for the littoral drift. The littoral drift mass-curve is the curve obtained by plotting (in a right-angled coordinate system) the distance from the nodal point (as abscissa) against the cumulative amount of erosion over this distance (5).

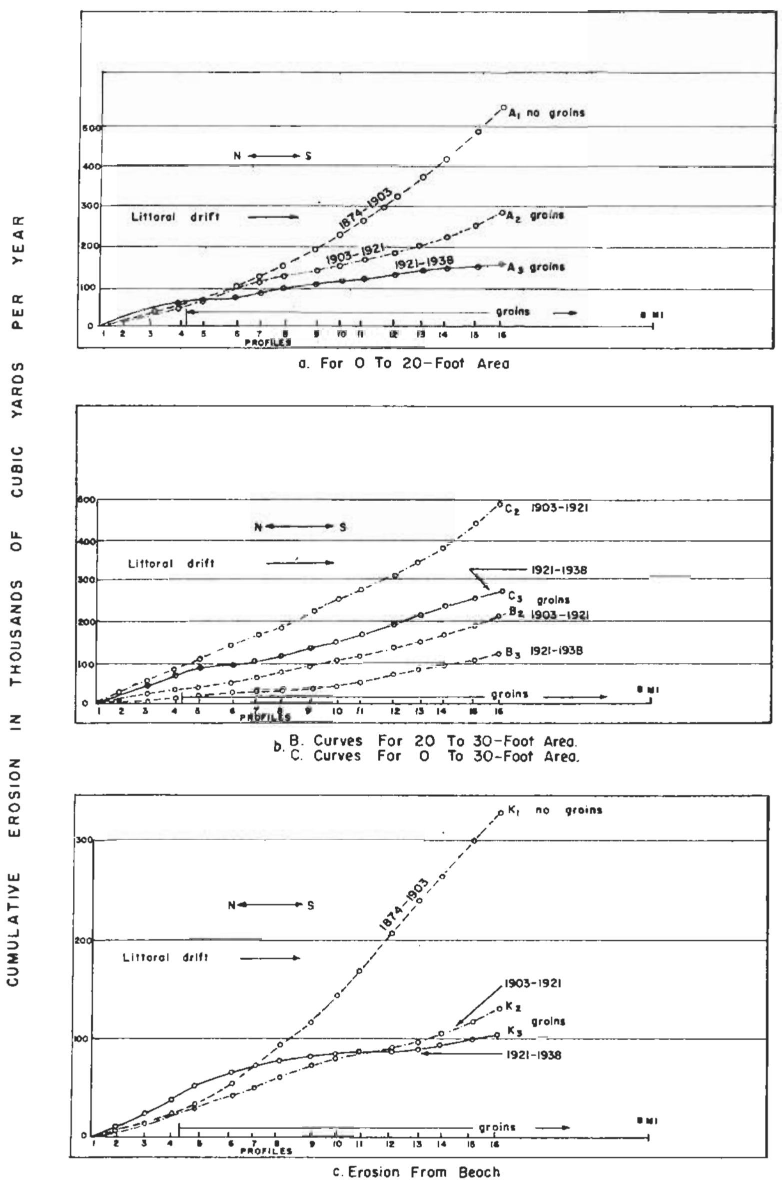


FIGURE 14. MASS CURVES FOR NORTHERN BARRIER

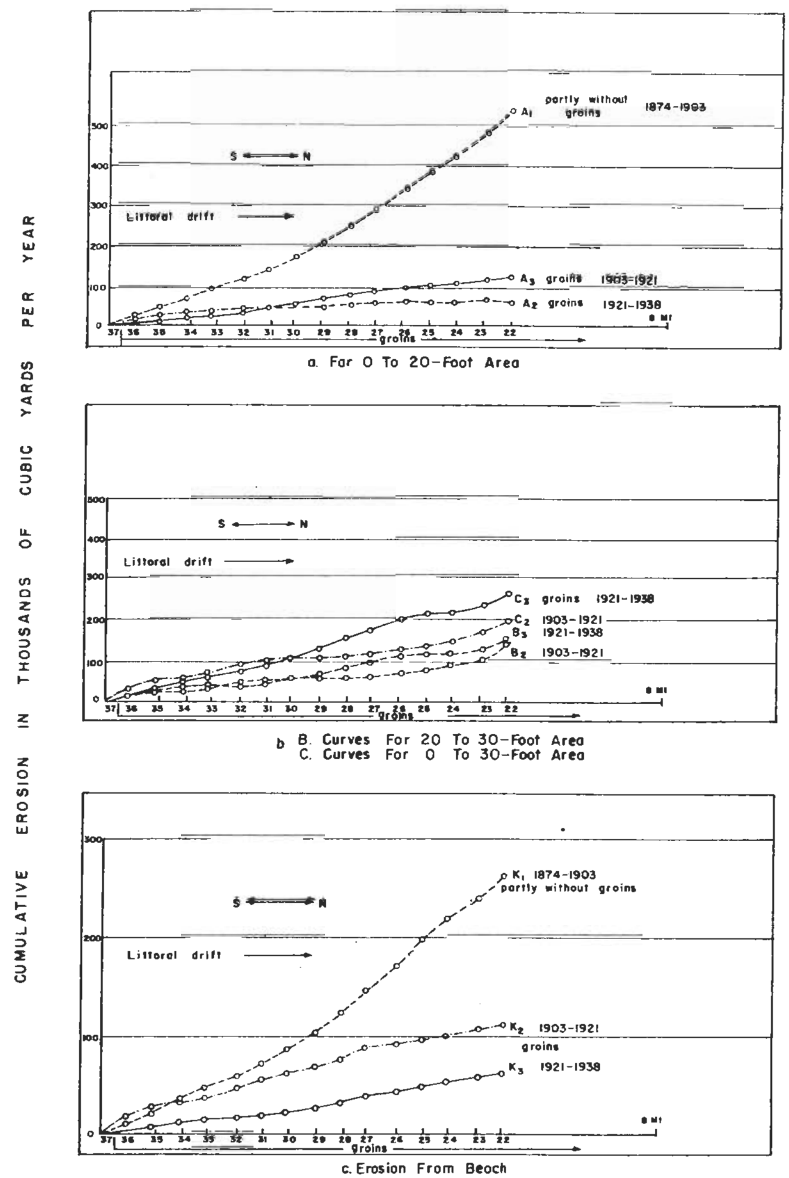


FIGURE 15. MASS CURVES FOR SOUTHERN BARRIER

The condition under which the shore line changes may be explained by means of mass-curves is that the beach profiles are equilibrium profiles, i.e., they have maximum steepness and do not change with time. Under these conditions, for a shore with a constant elevation, when the mass-curve is straight, the erosion is the same everywhere, i.e., the coast retains its shape. Since the mass-curves always start at a nodal point they also show the quantity of the nearshore littoral drift.

The steep beach profiles on the Lime Inlet Barriers probably first reached conditions of maximum steepness out to a 30 to 40-foot depth in 1927-1934 (perhaps earlier on the Southern Barrier, see sections 3.24 and 3.3). Therefore an investigation of the shore line changes by means of mass-curves relative to the 30-foot depth may give reliable results since the period 1927-1934, although it is possible that an investigation relative to the 20-foot depth would give reliable results for a longer period of time. The quantity eroded is calculated on the basis of depth soundings.

The first depth sounding was carried out in 1874. The spacing between profiles was 600 to 700 yards and soundings were taken every 16 yards out to a 20 to 30-foot depth. The next survey in 1897, was carried to a 30- to 40-foot depth. Since then soundings have been taken in 1899, 1901, 1903, 1905, 1907, 1909, 1911, 1913, 1916, 1921, 1923, 1925, 1927, 1929, 1931, 1934, 1936,1938, the depth measurements being 22 yards apart. World War II and its extensive mine fields essentially prevented the further accumulation of survey data, although in 1942 the coast between L5 and L8 on the Northern Barrier was sounded, as was (in 1950) that between L 31 - L 34 on the Southern Barrier. These soundings were obtained over a coastal reach of about 40 miles between L 29A (Vorupoer) and L61. In 1938 the sounded area was extended southward from L 61 to the southern part of Nissum Inlet (Figures 2 and 3).

The soundings generally were extended out to the 30 to 40-foot depth, but since 1936 every fifth line was sounded out to 60 to 70 feet. The nearshore soundings were obtained from a staff, and those in deeper water by lead line, although an echo sounder is now in use. These soundings were carried out only in the summer under calm weather conditions. The water level was obtained from tide gages at the groins.

The soundings along the Barriers have been used for calculation of the quantity eroded between L 1 and L 16 on the Northern Barrier (about 6 miles), and between L 22 and L 37 on the Southern Barrier (about 6 miles).

<u>The Northern Barrier.</u> Figures 14a and 14b show several different mass-curves. In all cases the distance, s, along the coast is measured southward from the nodal point L 1, and the quantity, B, is the cumulative erosion south of this point. It is assumed that the nodal point has always had the same position, and although this probably is wrong (see sections

3.12, 3.24, 3.25 and 3.4), a shift in the position of the nodal point will not materially change the form of the mass-curves. The percentage error in eroded quantities is given in Table 19, section 3.4.

Figure 14a shows mass-curves based on the annual erosion in the 0 to 20-foot area. A_1 is the mass-curve for 1874-1903, A_2 the mass-curve for 1903-1921, and A_3 that for 1921-1938. A_1 and A_2 show an immature shore line; A_3, however, is almost straight, with the exception of the northern part where the severest erosion now takes place and, therefore, seems to indicate maturity for the Barrier area.

Figure 14b shows mass-curves based on the annual erosion in the 20 to 30-foot area (indicated by B) and in the 0 to 30-foot area (indicated by C). B_2 and C_2 are the mass-curves for 1903-1921, B_3 and C_3 the corresponding curves for 1921-1938. C_3, like A_3, indicates a certain maturity. B_3 is almost straight between L9 and L 16, but,has not very far from the middle, a section (L5 - L9) which is almost parallel to the S axis. This possibly is due to the material from the beach and the 0 to 20-foot area, as a consequence of the increase in steepness, moving into the 20 to 30-foot area (see section 3.25).

The amount eroded above sea level is not taken in consideration since survey data were not obtained on the beach above this level. However, if the recession of the shore line is taken as representative of the erosion of the beach up to elevation plus 13 feet, the quantity eroded from the beach can be roughly calculated. In Figure 14c, K_1 is the mass-curve for this quantity for the period 1874-1903, K_2 for 1903-1921 and K_3 for 1921-1938. It may be seen that there is a correspondence between the shapes of the curves A_1 - A_3 (Figure 14a) and K_1 - K_3; A_3 and K_3 being almost straight in the same section of the coast, i.e., between L6 and L 16.

<u>The Southern Barrier.</u> Mass-curves based on the annual erosion in the 0 to 20-foot area are shown in Figure 15a. A_1 is, as before, the mass-curve for the period 1874-1903, A_2 for 1903-1921, and A_3 that for 1921-1938. L 37 is assumed to be the nodal point but as mentioned above, a shift in the position of the nodal point will not change the form of the mass-curves. A_1 shows immature conditions; A_2, almost straight between L 22 and L 34 indicates a certain maturity; A_3, which is almost straight over the entire area between L 22 and L 37 indicates maturity. A comparison with the conditions on the Northern Barrier shows that the Southern Barrier is the older of the two (see sections 3.24, 3.25, 3.3, 3.4, and 3.5).

In Figure 15b the corresponding curves C_2 and C_3 based on the erosion in the 0 to 30-foot area give almost the same impression as the curves A_2 and A_3 although both these curves fluctuate more than the A-curves and the corresponding C-curves on the Northern Barrier. Near the channel they turn upward but special conditions prevail here because of the current in the channel. Of the B-curves (based on the erosion in the 20 to 30-foot area), B_2 is almost parallel to the S-axis between L 27 and L 32 where the coast is very steep; B_3 perhaps indicates greater maturity, but both show fluctuations which must be supposed to indicate movement of material offshore, which again is a sign of maturity of the beach profiles (see section 3.25). Figure 15c shows the corresponding K-curves for the Southern Barrier, where as before, K_1 is for 1874-1903, K_2 for 1903-1921 and K_3 for 1921-1938. It may be seen that there is again an agreement between the shapes of the K- and A- curves (e.g., K_3 is almost straight over the same reach as A_3).

As previously mentioned, there is a correlation between the movement of the shore line and the erosion of the bottom, and it is quite understandable that this correlation is best when the 0 to 20-foot depth area is considered. It appears that on the Northern Barrier a section of the coast about 4½ miles long between L6 and L 16 has now attained an equilibrium form, while the severely eroded coast north of L5 is still in a transitional stage approaching the equilibrium form; on the Southern Barrier the 6-mile reach between L 22 and L 37, has attained equilibrium, but the coast south of L 37 is still in a state of change.

3.15 SUMMARY.

(a) The formation of Thyboroen Channel caused the adjacent coasts to become funnel-shaped. The influence area of the channel has gradually extended laterally. It appears as though the indirect channel influence runs from about Lyngby (L 14A) to about the northern end of Lake Ferring (L 48), possibly as far south as Bovbjaerg (see Figures 11a and 11b).

(b) The maximum annual shore line recession has apparently migrated from the channel and is now greatest in the unprotected areas at Lodbjaerg (about L 1) and south of Langerhuse (about L 40).

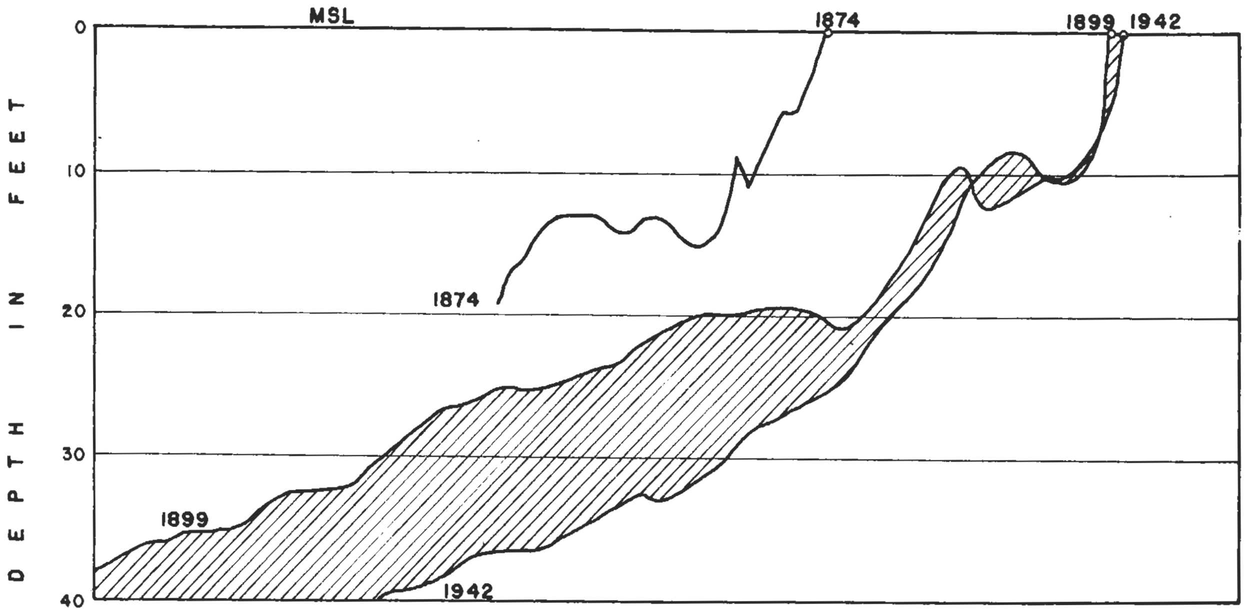
(c) A consideration based on "mass-curves" for the quantity eroded leads to the same result with respect to the extension of the channel influence area.

(d) The shore line recession on the Barriers has decreased considerably since the erection of groins. The minimum average occurred in 1927-1934. The same applies to the unprotected areas north and south of the Barriers and at Bovbjaerg, the protected area; but weather conditions were particularly favorable in this period (section 3.3). After 1927-1934 the shore line recession on the Barriers shows an increasing tendency. Present survey data do not permit definite conclusions about any increasing tendency north and south of the Barriers, although it probably exists.

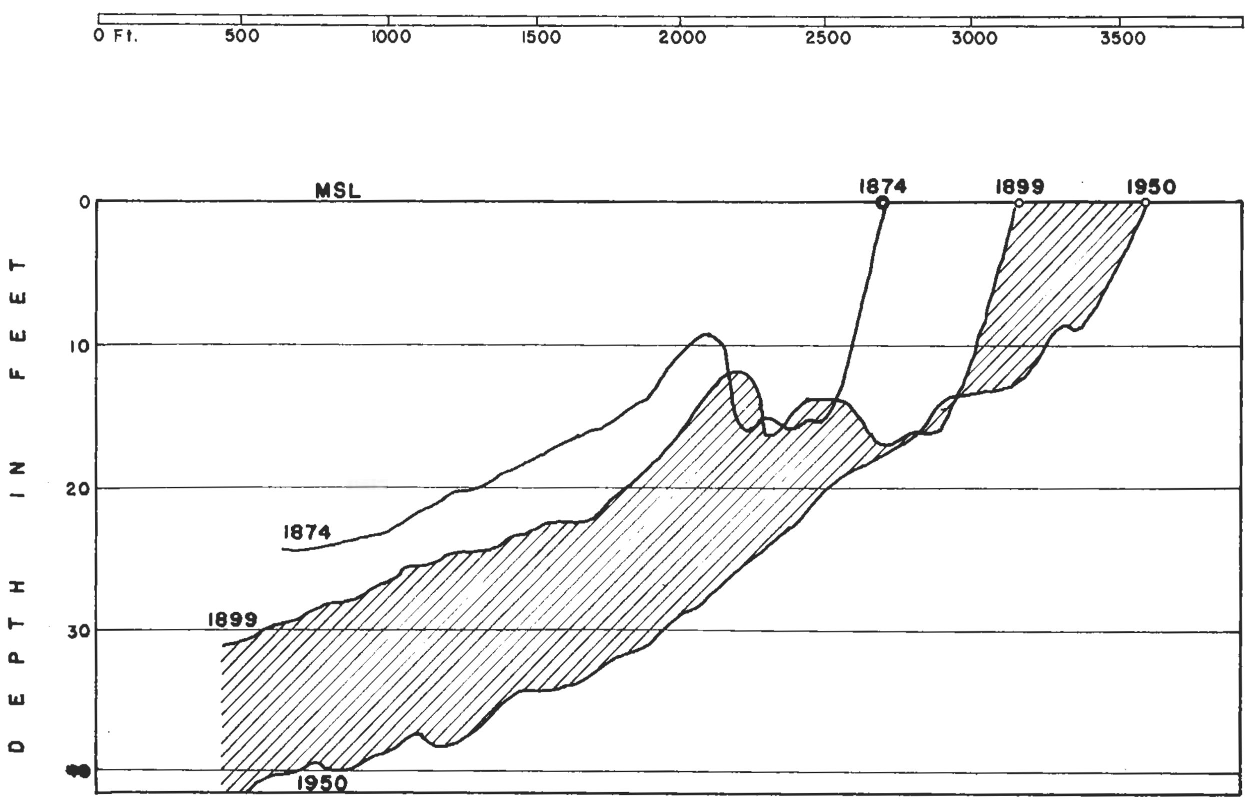
(e) The recession has gradually become more uniform in the Barrier areas proper, so that the configuration of the Barriers is not essentially changed.

(f) The shore line movement is of three separate types, migrating waves, seasonal fluctuations and long period changes due to erosion.

The movement of the entire beach profile due to erosion is less than the seasonal fluctuations are very small in comparison with the movements of the migrating wave type. Hence definite conclusions about the sea bottom scour on the basis of shore line movements and beach erosion should



(a) Profile L8 – Northern Barrier



(b) Profile L31 – Southern Barrier

FIGURE 16 BEACH PROFILES ON LIME INLET BARRIERS be drawn with caution, because the connection is partially obscured by seasonal fluctuations and migrating waves in the shore line. The damage done during the gale in October 1936 should presumably be viewed against this background and not regarded as evidence of "rush-outs" (see also sections 3.3, 3.52 and 3.53).

3.2 DEVELOPMENT OF THE BEACH PROFILE

3.21 Description of two characteristic profiles. Figure 16a shows beach profile L8 on the Northern Barrier in 1874, 1899, and 1942. Between 1874 and 1942 the shore line receded about 340 yards and the 20-foot depth contour about 450 yards; between 1899 and 1942 the shore line receded only about 20 yards, the 20-foot depth contour about 210 yards, and the 30-foot depth contour about 390 yards. This amount of recession, however, is not characteristic for that part of the Northern Barrier, a more representative amount being about 110 yards. Figure 16b shows beach profile L 31 on the Southern Barrier in 1874, 1899, and 1950. In the 1874-1950 interval the shore line receded about 310 yards and the 20-foot depth contour about 430 yards, between 1899 and 1950 the shore line and the 20- and 30-foot depth contours receded 155 yards, 225 yards and 430 yards, respectively, i.e., the profile has steepened while at the same time the bar almost disappeared.

3.22 Considerations on the basis of movements of depth contours. Table 5 shows the average rate of the recession of the shore line and the 20 and 30-foot depth contours calculated on the basis of measurements from L 1 - L 16 on the Northern Barrier and L 22 - L 37 on the Southern Barrier (advances being indicated by a minus sign)

TABLE 5 - AVERAGE RATES OF THE LANDWARD MOVEMENT OF THE 0, 20- AND 30-FOOT DEPTH CONTOURS (FEET PER YEAR)

	Northern Barrier Depth Contours			Southern Barrier Depth Contours		
Period	0'	20'	30'	0'	20'	30'
1874-1897	29	28		24	41	
1897-1903	21	55.5	21	7	39	-57
1903-1909	9.5	18	31	12.5	-11	73
1909-1916	7.5	15.5	43.5	3.5	- 1.5	12.5
1916-1921	10	45.5	26	7.5	4.5	12.5
1921-1927	9.5	3.5	19	6	28	27.5
1927-1934	4.5	16	39	1.5	21.5	57
1934-1938	4	-25.5	-42	6	-14	-75
1938-1948	11.5			6.5		

From Table 5 it may be seen that the recession of the shore line is less than the landward movement of the 20-foot depth contour, and that this again is less than the landward movement of the 30-foot depth contour. On the Northern Barrier, the annual landward movement of both the 20- and 30-foot depth contours decreased until 1927, after which a large annual recession and then a large annual advance occur; if the period 1927-1938 is taken as a whole, a continuously decreasing rate is found.

The movement of the depth contours on the Southern Barrier is less regular. From 1903 to 1916 the rate for the 20-foot contour is negative in two periods, then positive in three periods and negative again in 1934-1938. The rate of movement of the 30-foot contour decreased until 1916-1921, after which two periods with increasing tendency occurred, followed in 1934-1938 by a strong movement seaward. If the period 1921-1938 is taken as a whole the tendency is for an irregular but slightly decreasing change in rate of movement.

If the rate in one period of x years is given by V_1 and the rate in the next period of y years by V_2, then the rate of change or acceleration, is $\frac{2(V_2 - V_1)}{x + y}$.

Table 6 shows the accelerations corresponding to the rates shown in Table 5. A negative value indicates a decreasing rate (or deceleration).

TABLE 6 - ACCELERATION CORRESPONDING TO TABLE 5 (FEET PER YEAR^2)

	Northern Barrier Depth Contours			Southern Barrier Depths Contours		
Period	0'	20'	30'	0'	20'	30'
1897-1903 1874-1897	-0.6	1.9		-1.2	0.1	
1903-1909 1897-1903	-1.9	-6.2	1.7	0.9	-8.3	21.7
1909-1916 1903-1909	-0.3	-0.4	1.9	-1.4	1.5	-9.3
1916-1921 1909-1916	0.4	5.0	-2.9	0.7	1.0	0.0
1921-1927 1916-1921	-0.1	-7.6	-1.3	-0.3	4.3	2.7
1927-1934 1921-1927	-0.9	1.9	3.1	-0.7	-1.0	4.5
1934-1938 1927-1934	-0.1	-7.5	-14.7	0.8	-6.5	-24.0
1938-1948 1934-1938	1.1			0.1		

From Table 6 it may be seen that the acceleration of the shore line recession on the Northern Barrier in the main is negative (i.e., decelerating) until 1938, and on the Southern Barrier negative until 1934. On the Northern Barrier, the movement of the 20-foot depth contour shows mainly decelerations since 1921, as it does on the Southern Barrier after

1927; the 30-foot depth contour on the Northern Barrier has mainly decelerations after 1916; that on the Southern Barrier shows three small accelerations and two large decelerations irregularly distributed over the years after 1909.

If the period 1921-1938 is regarded as a whole decelerations wills be found everywhere for the 20-foot and 30-foot depth contours.

Similar investigations in other periods with the 27 and 33-foot depth contours show similar results. Thus it appears as though changes of the erosion rate of the shore line are apt to be accelerations, while those of the 20- and 30-foot depth contours are more apt to be decelerations. It is, however, possible that the erosion rate of the 20-foot depth contour will accelerate a little together with the shore line.

The conclusion from this would seem to be that the beach profile is adjusting itself into an "equilibrium profile" with maximum steepness (see sections 3.23 and 3.72).

3.23 <u>Calculations on the basis of the beach profiles.</u> The cross sectional water area corresponding to the beach profiles: The profile area is defined as the area lying between the mean water level, the sea bottom, and a vertical line at the depth contour.considered. This area between the 0 and 20, 0 and 30, and 20 and 230 foot depth contours has been calculated for the Barrier profiles L 1 - L 16 and L 22 - L 37 for the years 1874, 1897, 1903, 1916, 1921, 1927, 1934, and 1938. In addition calculations have been made at L5-L8 for 1942 on the Northern Barrier and at L 31 - L 34 on the Southern Barrier for 1950. Similar calculations were made for profiles north and south of the Barriers (L 1A - L 14A and L 38 - L 61) for the years 1874 or 1876, 1903, 1921, 1927, 1934, and 1938.

Special investigations were made on two 550-yard (500 m) wide fixed areas of the bottom along the Barriers as mentioned in sections 3.25 and 3.4.

Calculations on the basis of four separate surveys of the same beach profile, as well as consideration of the method of determining the area, show that the area determination is quite accurate, the standard deviation being of the order 0.5 to 1 percent.

Table 7 lists the average areas between the 0 to 20-foot, 20 to 30 and 0 to 30-foot contours on both Barriers. From this table it may be seen that the minimum values always occurred in 1934; detailed investigations of the single profiles show a similar result, although some irregularities appear.

TABLE 7 - AVERAGE PROFILE AREAS (SQUARE YARDS)

	Northern Barrier Interval			Southern Barrier Interval		
Period	0 to 20'	20' to 30'	0 to 30'	0 to 20'	20' to 30'	0 to 30'
1874	2100			2290		
1897	2170	3000	5170	1810	3520	5330
1903	1830	3580	5410	1460	5130	6590
1909	1850	3360	5210	1690	3850	5540
1916	1800	2900	4700	1820	3500	5320
1921	1510	3000	4510	1800	3400	5200
1927	1510	2780	4290	1610	3370	4980
1934	1390	2420	3810	1380	2700	4080
1938	1750	2600	4350	1510	3380	4890

Width of the beach profile: A profile width may be defined as the horizontal distance between the two depth contours (including the shore line) defining the portion of the profile which is of interest.

Tables 8 and 9 show the widths of the contour intervals 0 to 20 feet, 20 to 30 feet and 0 to 30 feet. From these tables it may be seen that the average minimum values on the two Barriers always occurred in 1934. Many of the beach profiles on the Northern Barrier and especially profiles L4, L5 and L6 were probably too steep in 1934 and therefore the steepness decreased from 1934 to 1938. The average widths of the 0 to 20, 20 to 30, and 0 to 30 foot intervals in 1934 were 335, 290, and 625 yards respectively, but if the very flat profile L 16 is omitted because of its nearness to the channel and (perhaps more important) its location in the outlet area of the former Agger channel (Figures 3 and 8), the average widths become 330, 275, and 605 yards. The standard variation of the average width is 1 to 3%, being greatest for the 20 to 30-foot area. If an approximate beach profile is given by the equation $y^{3/2} = 0.09x$ (where y is the water depth in feet, and x is the distance in feet from the shore line) the resulting widths for the 0 to 20, 20 to 30, and 0 to 30-foot intervals are 330, 275, and 605 yards respectively, (see section 3.72).

On the Southern Barrier the average widths of the 0 to 20, 20 to 30, and 0 to 30-foot areas in 1934 were 350, 330, and 680 yards, but if L 34 is omitted because of its marked irregularity the average widths become 345, 315, and 660 yards respectively. An approximate beach profile of $y^{3/2} = 0.082x$ (y and x in feet) would result in widths of 360, 300, and 660 yards respectively (see section 3.72).

These "equilibrium profiles" are used in section 3.7 in an attempt to forecast the future recession of the shore line. In section 3.4 are mentioned some very long low sand waves which migrate along the coast in the direction of the littoral drift and which are, perhaps, caused by (longshore) currents (15, 22). Their length varies between 1,500 and 3,000 yards. These sand waves appear in Tables 8 and 9 as periodic

TABLE 8 - AVERAGE PROFILE WIDTH (CONTOUR SPACING) ON THE NORTHERN LIME INLET BARRIER (YARDS)

Year	Contours (feet)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Avg.	Ratio 0-20' / 20-30'
1874	0-20																		
	20-30	465	448	448	317	470	503	394	377	416	405	481	530	629	672	722	722	500	
	0-30																		
1897	0-20	454	448	459	372	421	437	492	416	410	481	514	618	547	662	635	623	501	
	20-30	246	202	257	410	262	257	164	333	301	355	334	443	645	388	608	689	369	1.36
	0-30	700	650	716	787	683	694	656	749	711	836	848	1061	1192	1050	1243	1312	870	
1903	0-20	345	388	426	334	350	301	394	345	356	361	471	547	569	553	569	601	431	
	20-30	394	235	236	378	280	274	230	388	575	548	586	619	258	537	657	739	433	1.00
	0-30	739	623	662	712	640	575	624	733	930	909	1057	1166	827	1090	1226	1340	864	
1909	0-20	416	437	334	350	273	257	361	383	361	394	421	426	514	612	569	558	417	
	20-30	262	350	344	323	355	306	230	301	394	503	416	525	328	394	678	820	408	1.02
	0-30	678	787	678	673	628	563	591	684	755	897	837	951	842	1006	1247	1378	825	
1916	0-20	503	328	405	328	394	399	426	355	383	426	405	405	448	503	448	416	411	
	20-30	262	306	284	350	186	230	208	295	224	405	339	569	197	426	569	634	343	1.20
	0-30	765	634	689	678	580	629	634	650	607	831	744	974	645	929	1017	1050	754	
1921	0-20	437	361	350	350	197	323	355	317	366	372	323	350	448	416	328	290	349	
	20-30	186	219	197	252	394	241	191	230	323	306	388	514	350	481	831	848	372	0.94
	0-30	623	580	547	602	591	564	546	546	689	678	711	864	798	897	1159	1138	721	
1927	0-20	317	339	394	186	328	328	383	350	306	372	328	284	383	394	339	459	345	
	20-30	372	219	208	284	208	109	159	350	405	366	339	503	465	416	525	547	345	1.00
	0-30	689	558	602	470	536	437	542	700	711	738	667	787	848	810	864	1006	690	
1934	0-20	426	383	350	284	295	224	339	328	295	350	361	295	328	328	361	372	335	
	20-30	170	219	219	213	230	257	241	279	262	186	273	383	295	383	426	580	290	1.15
	0-30	596	602	569	497	525	481	580	607	557	536	634	678	623	711	787	952	625	
1938	0-20	448	426	416	295	361	383	394	251	361	388	328	350	405	388	394	426	385	
	20-30	317	241	219	339	268	224	208	361	252	295	235	284	388	443	525	536	315	1.22
	0-30	765	667	635	634	629	607	602	612	613	683	563	634	793	831	919	962	700	
1942	0-20					301	317	334	268										
	20-30					273	230	230	186										
	0-30					574	547	564	454										

TABLE 9 - AVERAGE PROFILE WIDTH (CONTOUR SPACING) ON THE SOUTHERN LIME INLET BARRIER (YARDS)

Year	Contours (feet)	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	Avg.	Ratio 0-20' / 20-30'
1874	0-20																		
	20-30	705	705	651	590	569	498	492	525	481	465	432	514	481	569	574	558	550	
	0-30																		
1897	0-20	569	498	454	437	394	394	344	394	366	410	405	443	410	426	328	394	417	
	20-30	754	689	459	312	519	454	350	344	514	503	208	284	393	399	514	186	430	0.97
	0-30	1323	1187	913	749	913	848	694	738	880	913	613	727	803	825	842	580	847	
1903	0-20	470	410	317	328	317	301	317	262	273	306	350	421	448	437	388	448	362	
	20-30	1149	608	667	667	684	635	590	673	645	651	421	498	558	373	618	504	624	0.58
	0-30	1619	1018	984	995	1001	936	907	935	918	957	771	919	1006	810	1006	952	986	
1909	0-20	372	394	350	350	372	372	394	372	339	372	328	426	448	580	481	492	403	
	20-30	700	503	569	448	459	284	481	416	503	416	459	481	426	350	361	383	452	0.89
	0-30	1072	897	919	798	831	656	875	788	842	788	787	907	874	930	842	875	855	
1916	0-20	350	394	405	383	372	481	459	426	372	437	416	454	481	503	470	428	426	
	20-30	569	405	470	394	323	536	525	492	241	306	525	383	350	399	558	240	416	1.02
	0-30	919	799	875	777	695	1017	984	918	613	743	941	837	831	902	1028	668	842	
1921	0-20	459	416	448	383	372	405	421	448	383	416	405	448	426	536	448	579	434	
	20-30	405	459	437	339	437	416	465	454	426	437	219	328	410	427	437	366	406	1.07
	0-30	864	875	885	722	809	821	886	902	809	853	624	776	836	963	885	885	840	
1927	0-20	361	405	350	350	350	350	350	350	394	317	361	448	481	459	492	437	394	
	20-30	416	317	405	492	514	454	416	405	448	547	284	350	437	503	219	306	406	0.97
	0-30	777	722	755	842	864	804	766	755	842	864	645	798	918	962	711	743	800	
1934	0-20	262	350	361	339	339	344	388	361	372	339	312	290	426	416	350	317	350	
	20-30	394	235	361	426	383	224	257	350	109	219	339	470	454	219	372	416	330	1.06
	0-30	656	585	722	765	722	568	645	711	481	558	651	760	880	635	722	733	680	
1938	0-20	301	372	394	399	372	405	405	372	350	317	361	306	350	405	426	339	367	
	20-30	328	350	372	443	262	306	383	394	448	372	394	547	503	503	416	487	407	0.90
	0-30	629	722	766	842	634	711	788	766	798	689	755	853	853	908	842	826	774	
1950	0-20										372	328	328	394					
	20-30										186	361	394	459					
	0-30										558	689	722	853					

fluctuations in the width of the bottom areas on both Barriers. More detailed investigations of these waves are mentioned by the author in a paper to be presented at the Fifth Conference on Coastal Engineering in Grenoble, France, in September 1954.

3.24 <u>Development of mean depth and the steepness characteristic of the beach profile.</u> The mean depth "md" is the average depth of the water area over that portion of the beach profile that is of interest. It is the depth obtained by dividing the profile area A by the width b (26), as shown on Figure 17.

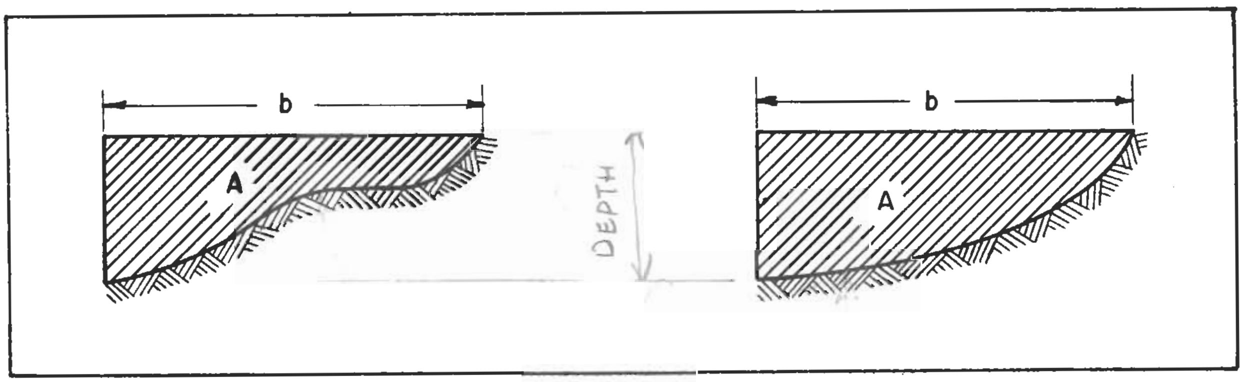


FIGURE 17 - $m_d = \frac{A}{b}$; $stc = \frac{m_d}{b} = \frac{A}{b^2}$

Fig 2. Steepness Characteristic.

The steepness characteristic of the beach profile "stc" is the mean depth divided by the width of the beach profile, md/b, or A/b^2 (26), see Figure 17.

Thus the steepness characteristic is a steepness parameter. A more practical value for the steepness of the beach profile may be obtained by multiplying the steepness characteristic by a factor of 2, because the mean depth is situated in the middle of the profile.

The steepness characteristic is introduced in order to differentiate between profiles of different shape which, however, have their limiting depths located at the same distances from the shore. For example, in Figure 17, the profile on the right is much steeper than that on the left.

Calculations of the standard variation of md and stc from repeated soundings of a single beach profile show these to be of negligible magnitude.

If the md of the single profile is taken as being representative of that over a 600-yard distance on either side of the particular profile, the standard deviation in the determination of the average md is about 7 percent for the 0 to 20-foot area, 3 percent for the 20 to 30-foot area and 2 percent for the 0 to 30-foot area. The corresponding values for four profiles (about 2,400 yards) are 5, 1, and 0-percent respectively, and for 16 profiles (about 10,000 yards) 3, 0 to 1 and 0 to 1 percent respectively.

If the stc of the single profile is taken as being representative of that over a 600 yard distance to either side of the particular profile, the standard deviation in the determination of the average stc is about 6 percent for the 0 to 20-foot area, 15 percent for the 20 - 30-foot area and 5 percent for the 0 to 30-foot area. The corresponding values for four profiles (2,400 yards) are 3, 10, and 3 percent and for 16 profiles (10,000 yards), 2, 4 and 1 percent respectively.

The way in which the calculations are carried out for the single profiles may increase their inaccuracies by only negligible amounts.

Table 10 shows the history of the average md and stc on the Barriers. From this table it may be seen that the average md for the three depth intervals has been almost a constant, the maximum deviation from the average values never being greater than 1.5 feet (in the 0 to 30-foot area), which fact commands special interest.

The table also shows that the increase in steepness is much more pronounced on the Northern than on the Southern Barrier, i.e, that the Southern Barrier is older in development than the Northern. On both Barriers the profiles were steepest in 1934 (0 to 30-foot area).

A glance at the history of the 0 to 20-foot area, where values of the annual shore line recession are added to the table, shows a correlation between increase of steepness and recession of the shore line. The great increase in steepness in the 0 to 20-foot area on the Northern Barrier between 1916 and 1927-1934 is succeeded by an increase in recession rate from 5 to 10 feet a year before 1934 to about 15.5 feet a year in 1938-1942. On the Southern Barrier the large increase in steepness in the periods 1897-1903 and 1927-1934 is succeeded by an increase in recession rate in the periods 1903-1909 and 1934-1938. This condition is indicated much more strongly for both Barriers in the detailed tables from which Table 10 was prepared.

The history of the 20 to 30-foot area shows that the stc has increased much more on the Northern Barrier than on the Southern Barrier. The average maximum stc on both Barriers occurred in 1934.

The history of the 0 to 30-foot area is an integral of the history of the 0 to 20 and 20 to 30-foot areas. Again it may be seen that the average maximum steepness has increased more on the Northern Barrier, which shows that the Southern Barrier, as mentioned above, is older in development than the Northern.

Extensive tables, corresponding to Table 10 and too comprehensive for inclusion in this report, have been prepared to show the development of scour, md and stc for every profile on the Barriers. These tables show that the breach of the Barriers at Thyboroen caused a very strong erosion of the nearshore areas of the bottom on both sides of the channel(see

TABLE 10 - HISTORY OF THE AVERAGE VALUES OF md AND stc ON THE BARRIERS

Year	Md (feet) 0 - 20'	Md (feet) 20 - 30'	Md (feet) 0 - 30'	Stc x 10^3 0 - 20'	Stc x 10^3 20 - 30'	Stc x 10^3 0 - 30'	Recession (feet)
NORTHERN BARRIER							
1874	12.8			9.0			
1897	12.8	24.7	17.8	8.9	4.9	7.1	29
1903	12.8	24.7	18.7	10.2	4.2	7.5	21
1909	13.1	24.7	18.7	11.1	4.5	8.0	9.5
1916	13.1	25.3	18.4	10.7	6.1	8.4	7.5
1921	12.8	24.0	18.4	12.6	4.8	9.0	10
1927	13.1	24.4	18.7	13.2	5.6	9.4	9.5
1934	12.5	25.0	18.1	12.8	6.7	9.9	4.5
1938	13.8	25.0	18.7	12.0	6.2	9.1	4
1942-							15.5
1945-							6.5
Average (Md)	13.0	24.8	18.4				
Max dev. (Md)	0.8	0.8	0.6				
Stc Increase (1897-38)				3.1	1.3	2.0	
SOUTHERN BARRIER							
1874	12.8			7.8			
1897	12.8	24.7	18.7	10.3	4.2	7.6	2.4
1903	12.5	24.7	20.4	11.8	2.9	7.0	7
1909	12.5	25.7	19.4	10.5	4.4	7.6	12.5
1916	12.8	25.0	18.7	10.1	4.5	7.5	3.5
1921	12.5	25.3	18.7	9.9	4.6	7.5	7.5
1927	12.2	25.0	18.7	10.4	4.5	7.8	6.5
1934	11.9	24.7	18.1	11.5	6.1	8.9	1.5
1938	12.5	25.0	18.7	11.3	4.5	8.2	6
1942-							5.5
1945-							9.0
Average (Md)	12.5	25.0	18.9				
Max. dev. (Md)	0.6	0.7	1.5				
Stc Increase (1897-38)				1.0	0.3	0.6	

Tables 2, 20, and 21), as a consequence of which the beach profiles flattened out. At the same time material from both Barriers drifted towards the channel and was to some extent deposited in front of the channel in a large bar which further flattened the beach profiles in the immediate vicinity of the channel. After this the influence area of the channel broadened away from the channel on both sides, causing at first a decrease in steepness of the beach profiles, in particular of the 0 to 20-foot interval; later on, especially after the construction of groins on the Barriers, the beach profiles steepened again. It appears that the Southern Barrier is older in development than the Northern because the increase in steepness is not only smaller but also more homogeneous there than on the Northern. Perhaps the changes described above can be generalized for the development of outlets in barriers.

The steepest beach profiles on the Northern Barrier are found between L 4 and L 9; on the Southern Barrier between L 30 and L 32, where the increase in steepness has also been the greatest.

The most recent soundings on the Northern Barrier (14-L 8 in 1942) show that L 5, L 6 and L 7 are not as steep as they were in 1934-1938, whereas L 8 is steeper than ever before, the stc out to the 30-foot depth contour being 12.3 percent. On the Southern Barrier the most recent soundings (L 31-L 34 in 1950) show that L 31, L 32 and L 34 were steeper in 1934-1938, but L 33 is steeper than ever before, the stc out to the 30-foot depth contour being 8.6 percent.

The increasing steepness, or perhaps rather the fluctuation in steepness, means that probably the beach profiles out to the 30-foot depth are not very far from "the equilibrium profile", i.e. a stable profile with maximum steepness, aside from fluctuations from one seasonal period to another, (see sections 3.71, and 5.2-5.3).

Table 11 shows the history of the average md and stc on the coast north of the Barriers between L 1A and L 14A (see Figure 3). From this table it may be seen that the average md for the 0 to 20, 20 to 30, and 0 to 30-foot intervals has been almost constant, the maximum deviation from the average values being 1.5 feet (in the 0 to 20-foot interval). Moreover, the table shows that the increase in steepness is not very pronounced. The maximum steepness occurred in 1927 in the 0 to 20-foot interval, 1934 in the 20 to 30-foot and 0 to 30-foot intervals.

A glance at the development of the 0 to 20-foot interval, where some values of the average movement of the shore line have been added to the table, shows that there is a correlation between increase of steepness and the rate of recession of the shore line, the changes in steepness and recession rates being similar during the period 1903-1934. As mentioned above it appears that an increase in recession will occur at the same time or some time after an increase in steepness. This tendency is more definitely indicated in the detailed tables corresponding to Table 11.

The detailed tables show that in the Southern part of the coast in the period 1921-1938 there was a tendency for the beach profiles to

TABLE 11 - HISTORY OF THE AVERAGE VALUES OF md AND stc ON THE COAST, NORTH OF THE BARRIERS

L 1A - L 14A

Year	Md (feet) 0 - 20	Md (feet) 20 - 30	Md (feet) 0 - 30	Stc x 10^3 0 - 20	Stc x 10^3 20 - 30	Stc x 10^3 0 - 30	Recession (feet)
1876	12.5	24.4	18.1	12.0	5.3	9.1	
							2
1903	13.5	24.0	18.8	12.5	4.4	8.7	
							5.5
1921	13.8	24.7	18.8	13.1	6.0	9.8	
							8.5
1927	13.8	24.7	19.1	13.8	5.5	10.0	
							3.5
1934	14.5	24.7	18.8	13.5	6.5	10.0	
							9
1938	15.4	25.3	19.8	12.8	6.7	9.4	
Average (Md)	13.9	24.7	18.9				
Maximum (Md) deviation	1.5	0.7	0.9				
Stc Increase (1876-1938)				0.8	1.4	0.3	

TABLE 12 - HISTORY OF THE AVERAGE VALUES OF md AND stc ON THE COAST SOUTH OF THE BARRIERS

L 38 - L 61

Year	Md (feet) 0 - 20	Md (feet) 20 - 30	Md (feet) 0 - 30	Stc x 10^3 0 - 20	Stc x 10^3 20 - 30	Stc x 10^3 0 - 30	Recession (feet)
1874	13.8			9.1			
							6.2
1903	13.8	24.7	19.8	11.2	3.5	7.2	
							5.3
1921	14.5	24.7	19.8	10.3	3.5	6.9	
							8.9
1927	13.8	25.0	19.4	10.0	4.1	7.1	
							8.2
1934	13.5	24.7	18.8	10.2	4.6	7.7	
							12.5
1938	13.5	24.7	19.4	10.0	3.9	6.9	
Average (Md)	13.8	24.8	19.4				
Maximum deviation (Md)	0.7	0.2	0.6				
Stc Increase (1874-1938)				0.9			
(1903-1938)					0.4	-0.3	

flatten out, which probably indicates a broadening of the influence area of the channel (as mentioned for the Barriers).

Table 12 shows the history of the average md and stc on the coast South of the Barriers between L 38 and L 61 (Figure 3), which includes three different stretches of coastline; the unprotected coast between L 38 and L 47, the coast protected with groins between L 47 and L 57, and the unprotected coast between L 57 and L 61. From Table 12 it may be seen that the average md has been almost constant, the maximum deviation from the average values being 0.7 foot (in the 0 to 20-foot interval). Detailed tables corresponding to Table 12 show that the increase in steepness 1903-1938 is small or even negative (in the 0 to 30-foot interval) and that the maximum steepness out to a 30-foot depth always occurred in 1934.

There is no marked connection between increase in steepness and increase in rate of recession of the shore line even though the fluctuations on the coast between L 38 and L 47 are similar for the period 1921-1938.

The detailed tables corresponding to Table 12 show that in the northern portion in 1921-1938 there was a tendency for the beach profiles to flatten out, which again probably indicates an increase in the influence area of the channel (as mentioned for the Barriers and north thereof).

A comparison between the development of mean depths and steepness characteristics on the three different reaches is given in sections 3.51 and 3.52.

3.25 Sea bottom scour. The sea bottom scour is defined as the annual vertical erosion of the bottom, and is usually given in inches per year.

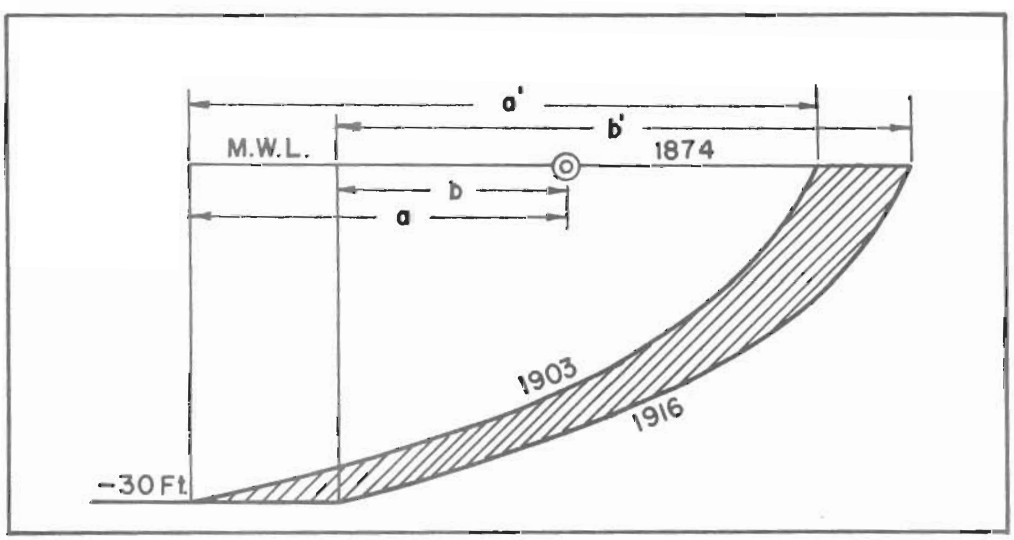


FIGURE 18 EROSION OF BEACH PROFILE

TABLE 13 - AVERAGE RATES OF SCOUR AND SHORE LINE RECESSION ON THE BARRIERS

Periods	Average Scour (inches per year) Intervals 0 - 20'	20 - 30'	0 - 30'	550-yd. Strip	Shore Line Recession Feet Per Year	Based on Measurements at:
			NORTHERN BARRIER			
1874-1897	4.6				29	16 points
1897-1903	5.5	4.3	4.9	4.4	21	16
1903-1909	3.6	2.2	2.9	2.3	9.5	16
1909-1916	2.0	4.3	3.1	2.6	7.5	36
1916-1921	3.7	0.4	2.2	2.9	10	36
1921-1927	1.0	2.2	1.6	2.3	9.5	36
1927-1934	1.8	4.8	3.2	2.8	4.5	36
1934-1938	3.8	-4.5	0.0	-2.6	4	36
1938-1942					15.5	32
1942-1945					6.5	32
1945-1948					12	
			SOUTHERN BARRIER			
1874-1897	5.2				24	16
1897-1903	3.9	-0.3	1.5	1.1	7	21
1903-1909	-0.3	3.3	1.9	1.5	12.5	21
1909-1916	1.4	-0.8	0.3	-0.2	3.5	21
1916-1921	0.6	1.1	0.8	0.8	7.5	21
1921-1927	2.4	2.3	2.3	2.4	6.5	21
1927-1934	1.3	3.9	2.6	3.2	1.5	21
1934-1938	0.7	-4.5	-1.9	2.7	6	21
1938-1942					5.5	20
1942-1945					9	20
1945-1948					4.5	

Figure 18 shows, diagramatically, two different beach profiles, one in 1903 and the other in 1916, placed on the graph in the proper relation to the location of the 1874 shore line. The area between the mean water level, the beach profile and a vertical line at the depth considered (for instance the 30-foot depth) may be computed for both profiles. If the area in 1903 is denoted by A, that in 1916 by B, and the distance between the 1874 shore line and the intersection point between the water level and a vertical line at the 30-foot depth contour by a and b respectively (Figure 18) the eroded area, S, is given by the equation S = B-A + 30 (a-b). (S having here the dimensions of square feet).

Calculations of the error in determining S for the single profile for the 0 to 20, 20 to 30, and 0 to 30-foot intervals show relative values between 1 and 2 percent. The error in the average scour of the whole sea bottom along the Barriers (as mentioned in section 3.4) is almost the same as the error in the eroded quantity (see Tables 18 and 19).

Table 13 shows the average scour of the bottom along the Barriers in the 0 to 20, 20 to 30, and 0 to 30-foot intervals, and in a 550-yard (500 m.) wide strip. This 550-yard wide area on the Northern Barrier was situated in 1897 between depths of about 7 and 23 feet, and in 1938 between depths of about 17 and 35 feet. On the Southern Barrier the corresponding depths in 1897 were about 13 and 27 feet, and in 1938 almost 20 and 33 feet. This special strip investigation results in a picture of the development which is independent of the landward movement of the beach profiles. The major conclusions from Table 13 are summarized below.

The development on the Northern Barrier: The construction of groins on the Northern Barrier in 1899-1909 caused a considerable decrease in shore line recession as well as in scour of the bottom in the 0 to 20-foot interval, as may be seen by comparing the area in 1903-1909 with that in 1897-1903. In addition, it may be seen that following a period of heavy erosion in the 20 to 30-foot interval, a strong decrease in scour took place (1916-1921) or even deposition occurred (1934-1938). This development obviously propagates from the 20 to 30-foot interval to the 0 to 20-foot interval, (as is shown by the development in 1909-1916-1921 and 1927-1934-1938) and then farther on to the beach. An increase in recession of the shore line ordinarily takes place in the same period as an increase in the scour of the 0 to 20-foot interval, (1916-1921), or more usually in the following period (see 1934-1938-1942). Simultaneously with an increase in the scour of the 0 to 20-foot area, the scour in the 20 to 30-foot interval decreases (see for instance 1916-1921), or even deposition takes place (see 1934-1938). This deposition in the 20- to 30-foot bottom interval can also be seen in the mass-curve B_3 in Figure 14b (see section 3.14). The 550-yard bottom strip shows characteristics similar to those of the 20 to 30-foot interval, but has more regular and moderate fluctuations.

Detailed tables show that the area of maximum scour of the bottom has migrated away from the channel, (see also sections 3.12, 3.14 and 3.4).

TABLE 14 - AVERAGE RATES OF SCOUR AND SHORE LINE RECESSION OVER LONGER PERIODS

Periods	Average Scour (Inches Per Year) 0 - 20'	20 - 30'	0 - 30'	550-yd. strip	Shore Line Recession (ft. per yr.)
			NORTHERN BARRIER		
1874-1934	3.5				17
1874-1938	3.5				16
1897-1934	2.8	3.2	3.0	2.9	11
1897-1938	2.8	2.4	2.7	2.3	11
1903-1934	2.3	3.0	2.6	2.6	8
1903-1938	2.5	2.1	2.4	2.0	8
1909-1934	1.9	3.2	2.6	2.6	8
1909-1938	2.2	2.1	2.2	1.9	7
1916-1934	1.9	2.7	2.4	2.6	8
1916-1938	2.2	1.4	2.0	1.7	7
1921-1934	1.2	3.5	2.5	2.6	7
1921-1938	1.7	1.7	1.9	1.3	6
1927-1934	1.8	4.7	3.2	2.8	4
1927-1938	2.2	1.4	2.0	0.8	4
1934-1938	3.8	-4.4	0.1	-2.6	4
1938-1942					15
1942-1945					6
1945-1948					12
			SOUTHERN BARRIER		
1874-1934	3.0				13
1874-1938	2.8				12
1897-1934	1.6	1.6	1.6	1.5	6
1897-1938	1.5	1.0	1.2	1.1	6
1903-1934	1.1	2.0	1.6	1.6	6
1903-1938	1.1	1.2	1.2	1.1	6
1909-1934	1.4	1.7	1.5	1.6	4
1909-1938	1.3	0.8	1.1	1.0	5
1916-1934	1.5	2.6	2.0	2.3	5
1916-1938	1.3	1.3	1.3	1.4	5
1921-1934	1.8	3.1	2.4	2.8	4
1921-1938	1.5	1.3	1.4	1.5	4
1927-1934	1.3	3.9	2.6	3.2	1
1927-1938	1.1	0.9	0.9	1.1	3
1934-1938	0.7	-4.5	-1.9	-2.7	6
1938-1942					5
1942-1945					9
1945-1948					5

The development on the Southern Barrier: The construction of groins on the Southern Barrier in 1875-1899 (mainly 1886-1892) caused a considerable decrease in recession of the shore line as well as in bottom scour in the 0 to 20-foot interval in 1897-1903 as compared with that in 1874-1897. In 1903-1909 it even seems as though a small deposit occurred in the 0 to 20-foot interval. It may be seen that the development on the Southern Barrier is very similar to that on the Northern. When in some years the erosion in the 20 to 30-foot interval has been considerable in the following years a strong decrease in scour or even deposition occurs (1897-1903, 1909-1916, and 1934-1938). This development obviously propagates from the 20 to 30-foot to the 0 to 20-foot interval (1903-1909-1916) and then farther on to the beach. An increase in shore line recession takes place in the same period as increase in the scour of the 0 to 20-foot interval (1903-1909) or -- more pronounced -- in the following period (1927-1934-1938). The deposition in the 20 to 30-foot interval can also be seen in the mass-curve B_3 in Figure 19 (see section 3.14) In 1909-1916 a proportionally great increase of the scour in the 0 to 20-foot interval was accompanied by deposition in the 20 to 30-foot interval.

The 550-yard bottom strip shows a similar development to the 20 to 30-foot interval, but, as on the Northern Barrier, with more regular and moderate fluctuations. Detailed tables show that the area of maximum bottom scour has migrated away from the channel (see sections 3.12, 3.14, and 3.4).

The soundings between L 31 and L 34 in 1950 show a continuous strong scour to have occurred since 1916 in the 20 to 30-foot interval, and this has caused an increase in the recession of the corresponding shore line from 5.6 feet in 1916-1938 to 12.2 feet.in 1938-1945.

Average scour on the Barriers over longer periods; Table 14 shows the average annual scour on the Barriers and also some average recessions of the shore line calculated for greater periods, having either 1934 or 1938 as terminal dates. The reason for the particular termination date is the special development in 1934-1938 where a large bottom deposition occurred along the Barriers (see sections 3.4 and 3.54).

On the Northern Barrier the scour in the 0 to 20-foot interval shows a decreasing tendency until 1927-1934. In 1934-1938 a considerable increase in the scour in the 0 to 20-foot interval occurred, along with a large deposit in the 20 to 30-foot interval. On the Southern Barrier the scour fluctuated, having a minimum value in 1934-1938.

On both Barriers the development of the 20 to 30-foot interval corresponds closely to that of the 550-yard strip mentioned below.

The development of the 0 to 30-foot area on the Northern Barrier is quite uniform, with fairly equal rates of scour except for the period 1934-1938. The Southern Barrier shows a uniform rate of scour if 1938 is used as the terminal date for the calculations; but if 1934 is used there is an increasing tendency as a consequence of the strong scouring in 1927-1934. In 1934-1938 deposition took place.

As regards the 550-yard strip on the Northern Barrier, it may be seen that while the scour rates calculated with 1934 as the terminal year are between 2.6 and 2.9 inches per year, the corresponding rates calculated on a 1938 basis show a uniformly decreasing tendency, culminating in a period of deposition. in 1934-1938. At the same time the rate of shore line recession increased to about 10 feet a year in 1934-1942 being about 5 feet a year before that time. This probably indicates a movement of material out from the shore as a consequence of the increase in steepness (see also sections 3.14 and 3.24).

On the Southern Barrier the annual scour rates calculated on the basis of 1934 increase from 1.5 inches in 1897-1934 to 3.2 inches in 1927-1934, while the corresponding values for 1897-1938 and 1927-1938 are 1.1 inches in both cases; however, a deposition of 2.7 inches per year occurred in 1934-1938. This, as mentioned above, may be due to movement of material out from the shore as a result of the increase in steepness although there may be other reasons.(see sections 3.54 and 5.3).

Since construction of the groins the average shore line recession on the Barriers has been about 8 feet a year on the Northern Barrier and about 6 feet a year on the Southern Barrier. Since 1934-1938 however, there has been a tendency to increasing recession (Table 13).

A peculiarity of Table 14 is that the average beach profile for 1934 on both Barriers may be obtained by a vertical shifting of the 1897 profile because the annual scour rates from 1897 to 1934, in all areas investigated, were remarkably constant, being in between 2.8 and 3.2 inches. on the Northern Barrier and between 1.5 and 1.6 inches on the Southern Barrier. As mentioned previously it appears as though the processes of shore line development are such that when the beach profile is steeper than a certain "equilibrium profile", material will move seaward (see section 3.14). The movement obviously takes place not as "rush-outs" but as a slow migration over the entire shore. In 1934-1938 this transverse movement of material may have reached a maximum though, as mentioned above, there is some reason to believe that in 1934-1938 material was carried to nearshore areas of the bottom from deeper water (see sections 3.54 and 5.3).

A comparison between the development of scour on different reaches (L 1 - L 14A, L 1 - L 16, L 22 - L 37, and L 37 - L 61) is mentioned in section 3.53.

3.3 <u>DEVELOPMENT OF THE BEACH PROFILE IN RELATION TO THE WIND CONDITIONS (WAVE CONDITIONS).</u> Since waves are mainly responsible for the development of the beach profile, as has been shown by laboratory and field experiments (2,3,6,12,20), knowledge of wave conditions is desirable. However, for the North Sea calculation of wave characteristics from known wind conditions is difficult, because that sea is shallow and spilling breakers,causing energy loss, occur to an extensive degree almost everywhere during storms. It is, therefore, not practicable to make a comparison between the erosion and the development of the beach profile and unreliable wave data. On the other hand, it may be possible to obtain a result by a statistical treatment of wind observations, although it must be remembered that the problems are complex and that results, if any, can only be accepted if statistical analysis indicates that they are probable. It is, therefore, not strange that attempts to correlate eroded quantities and wind conditions have failed.

The simplest investigations are those involving the development of the beach profile. As fluctuations in the beach profile along the Barriers as far as may be seen, will occur only when the profile has developed to a certain steepness, it seems probable that a connection between the fluctuations of the profile and wind conditions could be found only where the profiles are not far from their maximum steepness; that is, after 1934 on the Northern Barrier and after about 1909 on the Southern Barrier, which probably does not exclude the possibility of the profiles becoming somewhat steeper, especially in the reaches closest to the channel, where an increase in steepness is likely to occur. On the other hand results may possibly be attained from the development of the areas L 5 - L 8 on the Northern Barrier and L 31 - L 34 on the Southern Barrier, as both of these have had a relatively large steepness since 1909.

Investigations with beach profiles have shown that strong onshore gales cause the greatest changes in the beach profiles, flattening them out. These storms, therefore, are especially important.

Meteorological observations were obtained from Vestervig, situated on the west coast not very far from the Lime Inlet Barriers. Observations were made 3 times every 24 hours. Only wind velocities ≥ 6 Beaufort are of interest here, and of these observations more than 80 percent have directions between SW and NW; that is, that they are on the average perpendicular to the shore line.

Table 15 shows a comparison between the development of the stc for the Southern Barrier, L 5 - L 8 and L 31 - L 34, and the meteorological observations.* The table shows the number of observations per year from S through W to N with a Beaufort velocity ≥ 6. Observations of forces 6-7, 8-9, and 10-11 Beaufort are taken separately. The different velocities are not "weighted" in the following discussion because of our lack of knowledge in the detail of these problems.

The interpretation of the table may be seen from the following example for the 1927-1934 period. The average annual number of wind observations in the 1927-1934 period with force 6-7, 8-9, or 10-11 Beaufort is less than that for the adjacent periods; that is, the coast must have been attacked proportionally less by storm waves in the period considered than in the adjacent periods. Consequently, it can be expected that the profile was steepened, which in fact is the case on the Southern Barrier and at L 5 - L 8 and L 31 - L 34. In the period considered, for example L 5-L 8 steepened from 10.6 to 11.0 per thousand.

From the table it may be seen that there is basis for the assumption that there is a connection between wind (wave) and beach profile conditions, as periods with proportionally more and stronger storms coincide

*The standard deviation is about 0.01% for the Southern Barrier and about 0.02% for L 5 - L 8 and L 31 - L 34 (4 profiles).

TABLE 15 - COMPARISON OF STEEPNESS CHARACTERISTICS AND WINDS

Period	Stc x 10^3 L 5 - L 8 1.6 miles		Stc x 10^3 L 31 - L 34 1.7 miles		Stc x 10^3 Southern Barrier 6 Miles		(S through W to N) Wind Velocity (Beaufort) 6-7 Observations per year	8-9	10-11
1909-1916	10.0	9.3	7.9	7.5	7.6	7.5	66.8	14.3	2.5
1916-1921	9.3	10.9	7.5	8.1	7.5	7.5	36.8	8.0	0.4
1921-1927	10.9	10.6	8.1	7.9	7.5	7.8	41.7	11.5	1.3
1927-1934	10.6	11.0	7.9	8.9	7.8	8.9	39.0	4.6	0.2
1934-1938	11.0	10.0	8.9	8.4	8.9	8.2	49.2	9.7	0.0
1938-1942	10.0	11.0					53.4	9.8	0.3
1942-1950			8.4	8.8			88.0	17.7	0.7
1934-1942	11.0	11.0					51.3	9.8	0.2
1934-1950			8.9	8.8			69.5	13.7	0.5

TABLE 16 - ANNUAL EROSION (CUBIC YARDS)

Periods	Interval 0 - 20'	20 - 30'	0 - 30'	550-yard strip
		NORTHERN BARRIER		
1874-1897	650,000			
1897-1903	725,000	473,000	1,198,000	689,000
1903-1909	435,000	259,000	694,000	352,000
1909-1916	242,000	452,000	694,000	409,000
1916-1921	403,000	43,000	446,000	450,000
1921-1927	98,000	219,000	317,000	356,000
1927-1934	168,000	416,000	584,000	437,000
1934-1938	385,000	-368,000	17,000	-412,000
		SOUTHERN BARRIER		
1874-1897	711,000			
1897-1903	435,000	-47,000	388,000	174,000
1903-1909	-7,000	509,000	502,000	240,000
1909-1916	165,000	-96,000	69,000	-29,000
1916-1921	73,000	134,000	207,000	134,000
1921-1927	279,000	256,000	535,000	377,000
1927-1934	136,000	418,000	554,000	521,000
1934-1938	71,000	-473,000	-402,000	-425,000

with periods of decreasing steepness. In 1927-1938, L 1A - L14A and L 38 - L 61 show the same fluctuations. In 1921-1927 the fluctuation is reversed, but the change in steepness in this period is everywhere small and not very far from the error in the calculations made. On the Northern Barrier, which was flatter in 1897 than the Southern Barrier, the average steepness increased continuously from 1897-1934 (aside from smaller fluctuations inside the periods considered). This is especially caused by the steepening of the southern part, which was flattened out by the appearance of the channel. In 1934-1938 the steepness decreased.

Regarding conditions after 1938 it must be noted that a continuous strong wave attack following a great decrease in steepness (as in 1938) may increase the steepness a certain degree when the beach profile has been "too flat". As mentioned in section 3.54 there is reason to believe that storm waves intermittently bring in material from deeper water, which means that the seasonal fluctuation is possibly a more complex phenomenon consisting in part of a flattening of the nearshore profile, and in part of a flattening of the profile in deeper water (the 20 to 30-foot area), because storm waves, as far as can be seen, can bring in material from deeper areas of the sea bottom, provided that such material is available.

As mentioned above it has not been possible to prove any relation between quantities eroded and wind/waves.

3.4 THE QUANTITY ERODED ALONG THE BARRIERS The quantities eroded or deposited are shown in Table 16. These quantities can be calculated if the scour on the single beach profile is taken as being representative of the erosion for a certain area on both sides of the profile. It is very important to evaluate the reliability of a calculation such as this. This will depend on local conditions, as for example erosion, character of material, and the time interval considered. Undoubtedly it also depends on the migrating sand waves on the bottom. Investigations on the Danish North Sea coast show that the very long sand waves on the bottom migrate in the direction of the littoral drift (section 3.23 and (25) p. 219). Since the soundings are made at about 600-yard intervals, it has been difficult to evaluate the error in the eroded quantity calculated on the basis of the scour in single profiles. However, it can probably be taken for granted that the erosion has not varied greatly on shorter stretches of the coast. With this assumption, and giving due consideration to the x^2 - distribution, Table 17 has been computed for both Barriers for a 6-year period. Undoubtedly the computed is greater than the actual error.

TABLE 17 - ERROR IN CALCULATIONS OF THE QUANTITY ERODED ALONG THE BARRIERS IN A 6-YEAR PERIOD (CUBIC YARDS).

Interval	0 to 20'	20 to 30'	0 to 30'
NB (about 6 miles)	350,000	330,000	450,000
SB (about 6 miles)	240,000	420,000	420,000

It may be noted that the errors for the 0 to 20-foot and 20 to 30-foot intervals are different for each Barrier. For the 0 to 20-foot interval this is due to the greater erosion on the Northern Barrier;

for the 20 to 30-foot interval the greater error on the Southern Barrier must be because that Barrier is "older" in development than the other (sections 3.24b, and 3.25), which causes a more irregular movement of material from the beach and nearshore areas seaward, resulting in a more irregular scour in the 20 to 30-foot interval and, therefore, a greater error.

Table 18 shows the percentage error in average quantity eroded and scour along the Barriers (about 2 x 6 miles) for different periods. A dash indicates an error greater than 50 percent, which occurs in periods of very small erosion. As the inaccuracy in the calculation of the bottom area between the shore line and the 20 or 30-foot depth contours, and between the 20 and the 30-foot contours is very small, Table 18, also indicates the error in the calculation of bottom scour (section 3.25).

The error in calculating the erosion in the area between two lines of soundings cannot be indicated generally, but on an average it is 2.5 to 3 times greater than on the entire Northern or Southern Barriers (16 profiles ~15 areas). The error in calculating the erosion of an area including four profiles is about 2.1 times as great (6-year period).

TABLE 18 - PERCENTAGE ERROR IN AVERAGE QUANTITY ERODED AND SCOUR OF THE SEA BOTTOM

	INTERVAL		
Periods	0 - 20'	20 - 30'	0 - 30'
	NORTHERN BARRIER		
1874-1897	2		
1897-1903	8	11	6
1903-1909	13	21	11
1909-1916	21	10	9
1916-1921	17		20
1921-1927		25	24
1927-1934	34	11	11
1934-1938	23	22	
	SOUTHERN BARRIER		
1874-1897	2		
1897-1903	10		18
1903-1909		14	14
1909-1916	21		
1916-1921			41
1921-1927	14	27	13
1927-1934	27	14	11
1934-1938		22	26

TABLE 19 - PERCENTAGE ERROR IN ERODED QUANTITIES CORRESPONDING TO THE MASS-CURVES IN SECTION 3.14

NORTHERN BARRIER

	0 - 20'	20 - 30'	0 - 30'
1874-1903	2		
1903-1921	6	7	4
1921-1938	11	12	8

SOUTHERN BARRIER

	0 - 20'	20 - 30'	0 - 30'
1874-1903	2		
1903-1921	18	14	10
1921-1938	9	14	7

Table 19 shows the percentage error in the long period erosion along the Barriers corresponding to the mass-curves of section 3.14. In fact the mass-curves should be provided with a shadow, the width of which enlarges with the length of the curve. If the error in the calculation of erosion of the single area corresponding to one profile is Δs, the width of the shadow should be $2\Delta s \sqrt{n - 1.5}$ where n is the number of profiles. From the table it may be seen that the errors are small.

Table 16 shows, as mentioned, the quantities eroded or deposited. From this table the following may be seen:

The Northern Barrier. The erosion of the 0 to 20-foot interval decreased in 1903-1909 after the construction of groins on the Northern Barrier in 1899-1909. The erosion shows a continuous decreasing tendency until 1927, after which it increases. The erosion of the 20 to 30-foot interval shows a continuous decreasing tendency until 1921. After this there is a tendency to greater fluctuations, and in 1934-1938 deposits took place, resulting probably from the nearness of profiles to their maximum steepness (section 3.24). The erosion of the 0 to 30-foot interval also has a continuous decreasing tendency, although after 1916-1921 it appears to stabilize at a value of about 400,000 cubic yards. The erosion in the 550-yard strip has been almost constant in the period 1903-1927. The deposits in 1934-1938 were, however, an abrupt change.

The Southern Barrier. The erosion of the 0 to 20-foot interval decreased in 1897-1903 after the construction of groins on the Southern Barrier, (mainly in 1886-1892). After this the erosion fluctuated, though having, since 1921-1927, an overall decreasing tendency. The erosion of the 20 to 30-foot interval fluctuates widely, showing no particular tendency, and this can probably be taken as a sign of maturity of the beach profiles (see section 3.24). The erosion of the 0 to 30-foot interval is similar. After two periods (1921-1927 and 1927-1934) having an increase in erosion, a considerable deposit takes place

(1934-1938). These fluctuations, as mentioned above, probably also can be taken as a sign of maturity. The average erosion seems to be between 250,000 and 350,000 cubic yards. The erosion of the 550-yard strip also shows similar conditions. As mentioned above, large deposits occurred in the 20 to 30-foot and 550-yards areas in 1934-1938. This is the case along the entire coast between L 14A and L 61 (about 25 miles), and the quantity amounts to about one million cubic yards a year. Even if the severe erosion of dunes and dikes in 1936 is taken into consideration, it is impossible to account for these large deposits. A possible explanation, however, may be a moving in of material from greater depths (see section 3.54).

TABLE 20 - AVERAGE ANNUAL EROSION IN THE 0 to 30-FOOT INTERVAL
(Cubic Yards per yard per year)

Periods	Northern Barrier
1897-1903	116
1903-1909	67
1909-1916	67
1916-1921	43
1921-1927	31
1927-1934	56
1934-1938	1
	Southern Barrier
1897-1903	37
1903-1909	48
1909-1916	7
1916-1921	20
1921-1927	51
1927-1934	53
1934-1938	-38

Table 20 shows the average annual erosion per yard of shore line in the 0 to 30-foot interval on both Barriers. More detailed tables (such as Table 21 for the 0 to 30-foot interval) show that the strongest erosion in the latest periods occurred on the unprotected coast outside the Barriers (north of L 5 on the Northern Barrier and south of L 35 on the Southern Barrier) which means that the influence area of the channel has enlarged on both sides (see sections 3.12, 3.14, 3.24, and 3.25).

TABLE 21 - QUANTITY ERODED IN THE 0 to 30-FOOT AREA
(Cu. Yards per yard per year)

Profiles

Periods	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16	Avg. L1-L16
1897-1903	-11	16	68	110	139	142	55	17	96	170	180	223	220	171	146	116
1903-1909	-18	26	61	61	75	70	74	100	54	35	77	47	85	127	111	67
1909-1916	26	20	5	31	10	1	23	61	86	92	117	160	120	103	162	67
1916-1921	129	139	90	93	122	102	68	-29	-5	28	-46	-77	-4	34	12	43
1921-1927	37	33	34	-10	-10	12	17	41	35	12	55	49	31	81	41	31
1927-1934	60	33	50	54	40	41	52	50	57	78	63	81	69	53	77	56
1934-1938	8	50	87	62	-25	14	-1	-14	-17	-14	12	-10	-19	-48	-60	1
1938-1942					49	34	68									
1897-1938	35	42	52	56	52	54	42	37	48	62	73	78	79	79	78	57

Periods	22-23	23-24	24-25	25-26	26-27	27-28	28-29	29-30	30-31	31-32	32-33	33-34	34-35	35-36	36-37	Avg. L22-L37
1897-1903	60	66	66	85	59	-1	2	80	110	65	13	1	18	-12	-49	37
1903-1909	134	81	67	29	55	49	25	28	37	32	28	41	-22	38	110	48
1909-1916	67	44	-11	16	-22	-44	-11	17	2	-20	-11	11	37	6	14	7
1916-1921	-74	-35	32	-14	16	86	13	-43	-6	53	84	51	8	78	49	20
1921-1927	130	78	42	49	18	22	110	73	44	54	26	19	48	35	32	51
1927-1934	67	48	37	43	85	89	48	56	68	37	31	53	44	35	63	53
1934-1938	-56	-90	-83	-45	-8	-36	-67	-37	-13	-30	-48	-25	-12	-10	-35	-38
1938-1942																
1938-1950										57	57	35				
1897-1938	56	35	26	26	31	24	22	31	38	28	18	24	20	24	29	29

3.5 COMPARISON OF DEVELOPMENT ALONG THE BARRIERS AND ADJACENT AREAS.

3.51 Mean Depths. Table 22 shows the md characteristics of different coastal stretches. The "average mean depth" is the average of the mean depths of all beach profiles in the year considered. The "average mean depth of the single beach profile" is the average of the mean depths of the profile considered, all measurements included. A mean depth of about 18 feet (out to the 30-foot depth) corresponds to that obtained from a 1.5^{o} parabolic profile, and one of about 20 feet to a 2^{o} parabolic profile (see Table 32).

From Table 22 it may be seen that the average md for the different reaches is between 18.4 and 19.7 feet; in 1934 when the steepest profiles occurred, it varied between 18.1 and 19.0 feet.

The confused conditions in the nearshore area do not permit any other conclusion than that the shore development as far as can be seen is such that the variation of the average md is very limited.

TABLE 22 - THE VARIATION IN MEAN DEPTH IN THE 0 to 30-FOOT INTERVAL (FEET)

	L1A-L14A	NB	SB	L38-L47	L48-L57	L58-L61
Avg. mean depth	18.9	18.4	18.9	19.4	19.7	19.0
Max. deviation of the avg. mean depth from the avg. in the single year	0.9	0.6	1.5	0.7	1.0	1.6
Max. deviation of the single profile from the avg. of the whole area	0.7	1.6	1.0	0.7	2.3	0.3
Max. deviation of the single profile from the avg. of the single profile	3.0	3.3	2.6	2.3	3.0	2.6
Average mean depth 1934	18.8	18.1	18.1	19.0	18.7	18.4

3.52 Steepness characteristics. Before a comparison between the steepness characteristics of the different coastal areas is carried out it must be determined if the bottom along the reaches considered is composed of the same material. Information from fishermen and investigations carried out with the help of divers show that in the area of interest the bottom everywhere is composed of, or covered by, sand to depths greater than 30 feet. Beneath the sand is clay of different sorts; very soft inlet-deposited clay along the Barriers, and moraine clay north and south of the Barriers. This clay is also eroded and, therefore, it must sometimes be uncovered during heavy gales, but after a storm it is again covered with sand and, therefore, it must be the sand which determines the form of the beach profile. Consequently a comparison on the same basis can be carried out between the different coastal reaches.

Table 23 shows the development of the steepness in the 0 to 30 foot area between 1921 and 1938, where the development has the greatest interest.

TABLE 23 - DEVELOPMENT OF STEEPNESS CHARACTERISTICS ON DIFFERENT COASTAL REACHES ($stc \times 10^3$)

	SECTION					
Years	L1A-L14A	L1-L16 (NB)	L22-L37 (SB)	L38-L47	L48-L57	L57-L61
1921	9.8	9.0	7.5	6.8	7.1	6.4
1927	10.0	9.4	7.8	7.2	7.2	6.5
1934	10.0	9.9	8.9	7.7	7.8	6.7
1938	9.4	9.1	8.2	7.0	7.1	6.2
Increase in Stc						
1921-1934	0.2	0.9	1.4	0.9	0.7	0.3
1921-1938	-0.4	0.1	0.7	0.2	0	-0.2
Max. Steepness to 20' (year)	1927	1927	1934	1903	1903	1903
Max. Steepness to 30' (year)	1927&1934	1934	1934	1934	1934	1903
Absolute Max. steepness to 30', single profile	12.7 (L11A)	13.5 (L4)	10.4 (L31)	8.6 (L42)	8.9 (L50)	7.6 (L60)

From Tables 10-12, 23 and more detailed tables it may be seen that: (a) In 1897 the Southern Barrier was the steeper of the Barriers; since then the Northern Barrier has been steeper. However, the unprotected area L1A-L14A has continuously been the steepest portion of the coast, although in 1938 the Northern Barrier was almost as steep and the Southern Barrier not quite so steep. The great steepness between L1A and L14A has not caused any strong or irregular recession of the shore line (Tables 3 and 11). (b) The increase in steepness in 1921-1934 was greatest on the Southern Barrier. On the Northern Barrier and the coast between L 48 and L57 the increase was the same, being about half that on the Southern Barrier. The increase in steepness between L1A and L14A and between L58 and L61 was small. (c) The increase in steepness in 1921-1938 was greater on the Southern Barrier; the increase on the Northern was almost the same as that between L38 and L47, but only about 1/5 that on the Southern Barrier. Between L1A and L14A and between L58 and L61 the increase in steepness was negative (i.e., the steepness decreased). (d) The maximum steepness out to the 20-foot depth was greatest in 1934 on the Southern Barrier, in 1927 on the Northern and between L1A and L14A and in 1903 between L38 and L61. (e) The maximum steepness out to 30-foot depth was everywhere greatest in 1934, except for the reach between L58 and L61, where leeside erosion has taken place since about 1927. Here the steepness was greatest in 1903. (f) The steepest profile occurred at L4, on the unprotected portion of the Northern Barrier. L31 (1934) was the steepest on the Southern Barrier, but about twenty profiles on the unprotected coast L1A-L14A have been as steep or steeper in different periods. The steepest profiles generally occurred in 1934, although the record steepness (L4 on NB) was in 1927. (g) There is no reliable connection between md and stc, even though md often decreases at the same time as stc increases, which means that there is a tendency for the profile to approach a straight line.

The fact that the increase in steepness since 1921 has been greater on the Southern than on the Northern Barrier seems to show that the development of beach profiles on the Northern Barrier is approaching that on the Southern Barrier.

3.53 <u>Sea bottom scours.</u> It appears from section 3.25 that comparable fluctuations in the beach profiles occur on the three coasts, seeing that a continuous erosion of the 20 to 30-foot area is either concomitant with or succeeded by increased erosion in the 0 to 20-foot area, the shore line recession increasing synchronously with or subsequent to the erosion in the 0 to 20-foot area. This transverse movement seems to be very sluggish and there is no tendency to "rush-outs". The magnitude of the average scour in the 0 to 20-foot, the 20 to 30-foot and the 0 to 30-foot areas on the six different coastal reaches considered can be seen from Table 24.

The scour in the 0 to 20-foot area is always less than approximately 5 inches. In 1921-1938 it was 2.7 inches between L1 and L14A, as against 2.0 inches on the Northern Barrier and 1.5 inches on the Southern Barrier.

In the 20 to 30-foot area the scour is everywhere less than about 5 inches, but some depositions occur, the greatest being 7 to 8 inches. The greatest scours occurred on both Barriers in 1927-1934, amounting to 4 to 5 inches.

In 1921-1938 the scour was 1.8 inches between L37 and L48, 1.7 inches on the Northern Barrier and 1.3 inches on the Southern Barrier.

The scour in the 0 to 30-foot area is everywhere less than about 3 inches. A few depositions occur, not exceeding about 3 inches. In 1921-1938 the scour was 1.9 inches on the Northern Barrier, 1.5 inches between L37 and L48, 1.4 inches on the Southern Barrier, and 1.0 inch between L1 and L14A.

TABLE 24 - SCOUR PER YEAR ON DIFFERENT COASTS (INCHES)

Period	L1 - L14A	L1 - L16(NB)	L22 - L37 (SB)	L37 - L48	L48 - L57	L57 - L61
			0 - 20-feet			
1874(76)-1903	0.8	4.8	4.9	1.2	1.7	2.1
1903-1921	0.6	3.0	0.7	1.2	-0.1	0.6
1921-1927	4.2	1.0	2.4	1.5	0.4	0.6
1927-1934	0.2	1.8	1.3	1.9	0.6	3.1
1934-1938	4.8	3.8	0.7	-0.2	1.1	-1.6
1921-1938	2.7	2.0	1.5	1.3	0.6	1.1
			20 - 30-feet			
1874(76)-1903	-0.5					
1903-1921	3.2	2.5	1.2	-0.1	-0.2	-0.4
1921-1927	0.4	2.2	2.2	2.8	1.9	2.5
1927-1934	1.5	4.7	3.9	2.4	2.4	2.6
1934-1938	-7.3	-4.4	-4.5	-0.7	-4.7	-4.5
1921-1938	-1.0	1.7	1.3	1.8	0.6	0.9
			0 - 30-feet			
1874(76)-1903	0.2*					
1903-1921	1.8	2.8	1.0	0.6	-0.2	0.0
1921-1927	2.4	1.6	2.3	2.2	1.1	1.6
1927-1934	0.7	3.2	2.6	2.1	1.7	2.8
1934-1938	-0.4	0.1	-1.9	-0.5	-1.9	-2.9
1921-1938	1.0	1.9	1.4	1.5	0.6	1.1

* L1A - L14A

It thus looks as though the extension of the influence area of the channel is most pronounced to the south, where the whole 0 to 30-foot area seems to be affected. To the north, especially the 0 to 20-foot area is influenced. Yet it is debatable whether the increased erosion north of L8A is caused by the channel.

Table 25 shows the maximum scours in the individual areas, 1874(76) to 1950.

TABLE 25 - MAXIMUM SCOURS PER YEAR (INCHES)

Bottom areas	L1-L14A	(NB) L1-L16	(SB) L22-L34	L37-L48	L48-L58	L58-L61
0 - 20-feet	9	9.5	8	8	7	5.5
20 - 30-feet	4	14	7.5	7	6.5	5.5
0 - 30-feet	5.5	8*	5.5	5.5	4	5

* the unprotected area to the north

From the table it may be seen that the maximum scour is between 4 and 14 inches, the variation being greatest in the 20 to 30-foot area. The greatest scours occur on the Northern Barrier and generally along the steep coast L5-L8, where the beach profiles fluctuate most and, as far as can be seen, are determined by weather conditions (see section 3.3.)

3.54 Quantities eroded. The quantities eroded from the different coastal areas offer very few similarities. A consideration of the 0 to 30-foot area shows that the similarity is limited to the fact that the erosion, aside from L1 - L14A, increased in 1921-1934, while in 1934-1938 the following annual changes occurred.

L1 - L14A	-70,000	cubic yards
L1 - L16 (NB)	+20,000	" "
L22 - L37 (SB)	-400,000	" "
L37 - L61	-500,000	" "
Total about	1,000,000	" "

By comparison with 1927-1934 the average shore line recession on the Barriers increased from 2.7 feet to 5 feet; between L1A and L14A from 3.5 feet to 9 feet, and between L38 and L57 from 3.5 feet to 10.5 feet. The increase in shore line recession may at the very outside explain an extra supply of about 0.3 million cubic yards but a more reliable evaluation of the quantity is very difficult. The severe gale in 1936 caused some damage to dikes and dunes and may have caused material to be drawn out to the littoral berm without major influence on the position of the shore line. Yet a balance can be struck only by an erosion of 20 to 25 cubic yards/yard per year of the 25-mile coast, but this is improbable. It is possible, therefore, that the large deposition, as mentioned in section 3.4, can only be understood as a result of shoreward migration of material from depths greater than 65 feet (compare with conditions on the Dutch North Sea coast described by Thierry and van der Burgt in (22) pp. 139-142), and the investigations in Mission Bay mentioned in section 5.3. Eaton (6) has described similar observations along the Californian coast. All these investigations give reason to believe that during winter storms (or at least in periods of comparatively higher waves) some material migrates towards the shore. It is therefore, interesting to see that the 1934-1938 period is rather stormy (see section 3.3, Table 15). Probably it is not the first time that such migrations have occurred (see section 3.4, Table 16, the 20 to 30-foot area in 1909-1916). This period also shows comparatively more frequent storms than the preceding and subsequent periods (see Table 15).

3.6 RELATION BETWEEN SHORE LINE RECESSION, BOTTOM SCOUR, AND WIDTH OF THE 0 TO 30-FOOT AREA.

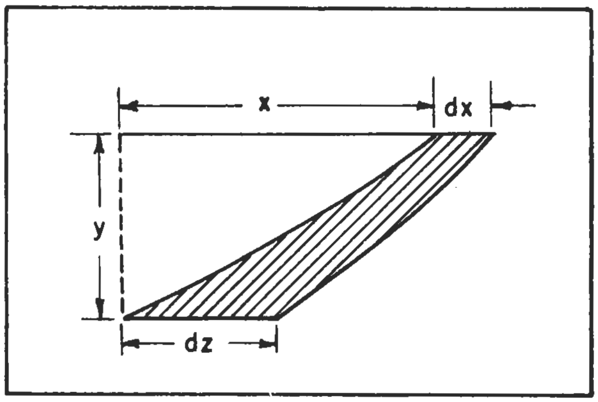


FIGURE 19 EROSION OF BEACH PROFILE

Figure 19 shows two different surveys (many years apart) of the same beach profile. In the time between these two soundings erosion has taken place. Under the assumption that a relation exists between shore line, recession, bottom scour, and development of steepness (as mentioned in sections 3.24 and 3.25) it seems probable that it would be possible to compute the movement, dz, of an arbitrary depth contour situated at distance x, from the shore line if the corresponding bottom scour, dn, and shore line recession, dx, are known. One obtains

$$y\left(\frac{dx + dz}{2}\right) = dn\left(x - \frac{dz}{2} + \frac{dx}{2}\right)$$

For example, to compute the movement of the 30-foot depth contour on the Northern Barrier in 1927-1934, the annual shore line recession, according to Table 2, is 4.5 feet, the annual scour, according to Table 13, is 3.2 inches or 0.26 foot, and the width of the 0 to 30-foot area in 1927 according to Table 8 is 2,070 feet. Then

$$\frac{(4.5 + dz)}{2} \cdot 30 = 0.266\left(2,070 - \frac{dz}{2} + \frac{4.5}{2}\right)$$

The last two numbers in this equation are very small (in comparison to the width, z,) and can be neglected. Then d_z equals 32 feet or, in seven years (1934-1927) about 225 feet. The recession of the shore line at the same time is 7 x 4.5, or about 30 feet. The width of the 0 to 30-foot area in 1934 therefore is 2,070 - 225 + 30 or 1875 feet, which is identical with that shown in Table 8 (section 3.23).

When the computations are carried out as described above for both Barriers for the period between the oldest deep-sounding in 1897 and 1934, the values indicated in Table 26 are obtained.

TABLE 26 - MEASURED AND COMPUTED WIDTHS OF THE 0 to 30-FOOT AREA

Year	Northern Barrier Width (ft.)	Southern Barrier Width (ft.)
1897 Measured	2,610	2,540
1934 Measured	1,875	2,040
1934 Computed	1,875	2,120

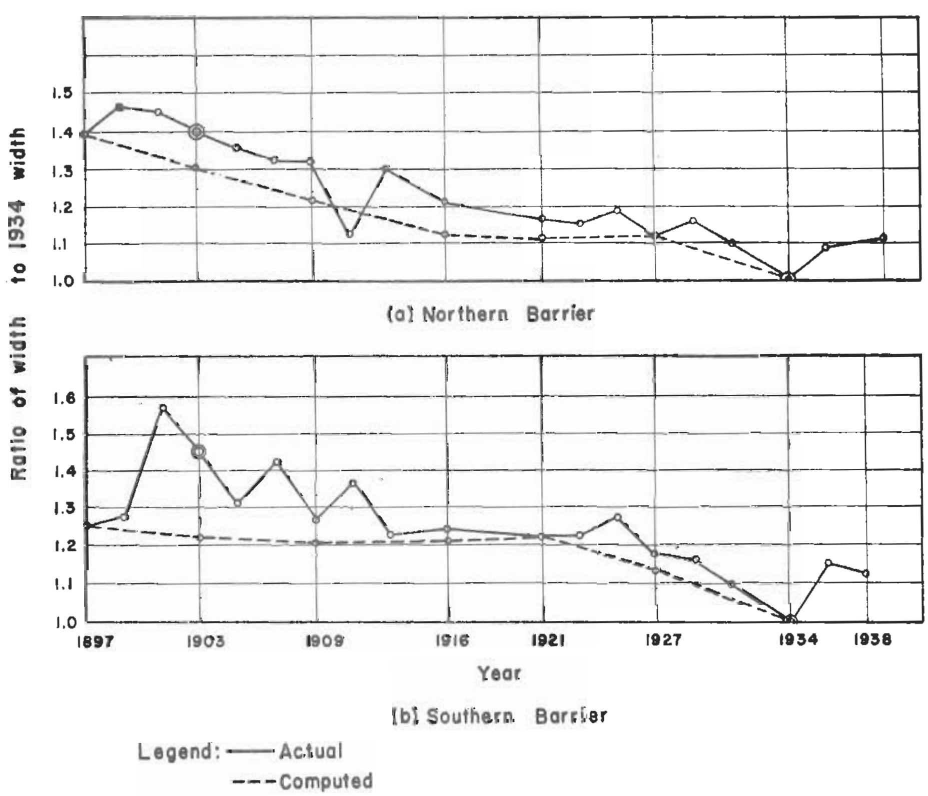


FIGURE 20 - WIDTHS OF 0 TO 30-FOOT AREA

If the average width along the Barrier is considered, the relative widths in proportion to those in 1934 are as indicated in Figure 20. Every survey made since 1897 is included. For the Northern Barrier there is good agreement at least in shape between the two curves. The development, aside from 1911, is remarkably regular. On the Southern Barrier there is good agreement after about 1913. Between 1901 and 1911 the beach profile fluctuates and is less steep, but the erosion continues (see Table 13). For both Barriers the agreement is best after 1909-1916, which tends to justify the use of the "equilibrium profiles" for evaluating the shore line recession (section 3.7).

3.7 USE OF EQUILIBRIUM PROFILES IN DETERMINING THE FUTURE DEVELOPMENT OF THE BEACH PROFILE AND THE RECESSION OF THE SHORE LINE.

3.71 Equilibrium profiles. An equilibrium profile is a beach profile which maintains its form. It is not known when this terminology was first introduced. Fenneman(8) writes: "There is a profile of equilibrium which the water would ultimately impart if allowed to carry its work to completion. The continual change of shore line and the supply of new drift are everchanging conditions with which no fixed form can be in equilibrium. There are, however, certain adjustments of current, slope and load which, when once attained, are maintained with some constancy. The form involved in their adjustments is commonly known as the profile of equilibrium. When this profile has once been assumed the entire form may slowly shift its position toward or from the land, but its slope will change little or not at all."

Perhaps there may be disagreement as regards the existence of an equilibrium profile when this profile is considered in detail. Saville(20) writes about experiments with equilibrium profiles in the laboratory: "In extending these experimental results to prototype conditions, it must be remembered that these results were obtained on so-called equilibrium beaches, beaches which were completely at equilibrium with the waves acting upon them. There was, therefore, no net transport of material perpendicular to the beach contour. Such a condition seldom if ever exists in nature." Saville is undoubtedly right as regards the development in detail, but Fenneman's theory deals with the general development of the "overall average form" as do the calculations mentioned below. The development of beach profiles is described by several other authors (2, 3, 7, 9, 12, 16, and 25). The most recent literature on this subject includes reports from the Beach Erosion Board, the University of California at Berkeley and Scripps Institution of Oceanography. (see also section 5.2 and 5.3.)

3.72 Geometrical form of the equilibrium profile. In section 3.23 an approximate beach profile was mentioned. An attempt to determine the bottom scour, dy, as a function of the shore line recession, dx, and the distance, x, from the shore line to the specified point at depth, y, where the scour is known, has for the steep profiles on both Barriers given the equation:

$$dy = \frac{P_1 \, dx}{x^{1/3}}$$

where P_1 is a constant which may vary from one point on the coast to another.

Integration gives by $y^{3/2} = Px$.

With x = 1815 feet and y = 30 feet, P = 0.09. This profile also passes through the points : x = 0, y = 0; and x = 990, y = 20. The actual average beach profile of the Northern Barrier for 1934 (except for L16) passes close to these same points (see section 3.23).

The ratio between the widths of the 0 to 20-foot and 20 to 30-foot areas on this equilibrium profile is 1.2 while in the actual average profiles this ratio is about 1.15 for 1934 and 1.22 for 1938.

With x = 1980 feet and y = 30 feet, then P = 0.082. This profile also passes through the points: x = 0, y = 0; and x = 1090, y = 20; while the actual average beach profiles on the Southern Barrier for 1934 pass through the points x = 0, y = 0; x = 1034, y = 20; and x = 1980, y = 30 (except for L34, see section 3.23).

The ratio between the widths of the 0 to 20 and 20 to 30-foot areas is 1.2, while in the actual average profiles this ratio is 1.06 for 1934, although the steepest profiles show greater ratios.

Naturally there are greater disagreements in the vicinity of and inside the bar. In depths greater than about 40 to 50 feet the actual profile will be below the theoretical and a parabola gives better agreement.

Figure 21 shows characteristic beach profiles, L8 on the Northern Barrier and L31 on the Southern Barrier, as sounded in 1938.

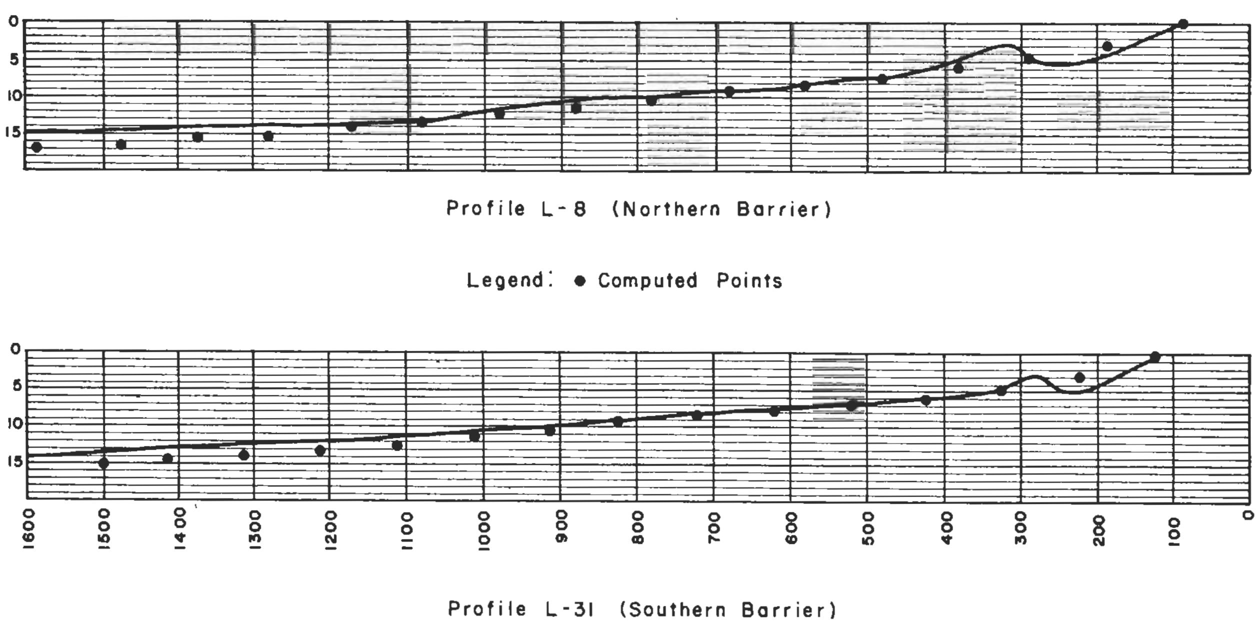


FIGURE 21-ACTUAL PROFILES AND COMPUTED POINTS

An attempt to approach the problem under certain very simple assumptions follows: A "one-bar" profile is to be considered. The section of the bottom seaward from the bar, from depths of about 16 to 45 feet, is about 3,500 feet in length, and the waves do not break in this area.

The assumptions for the calculations (I) described below are:

(I). a. The profile is formed by the shear stress due to the wave action and is at right angles to the shore line. The material detached by the oscillating water is removed by longshore currents. As the shear stress due to wave action in general- and partially during storms - is far greater than the shear stress originating from the longshore currents, this assumption seems logical.

b. In the equilibrium profile the shear stress per unit bottom area may be assumed to be constant, i.e., the "condition" at the bottom is the same $\left(\frac{d\tau}{dx} = \frac{d\tau}{dt} = 0\right)$

Confirmation of this assumption only can be attained by experiments. One obtains $\tau = K\rho\, u_{ave}^2$, where ρ is the density, K the resistance coefficient and u the water velocity. If τ is assumed a constant, then $u_{ave} \sim \frac{H\pi}{T \cdot \sinh \frac{2\pi y}{L}}$ is also constant where T is the wave period; H, the wave height; L, the wave length; and y, the water depth.

c. $\frac{dE_1}{dx}$ = constant, where E_1 is the transported wave energy per unit area of the wave, and x is the distance from the shore line. The loss of energy is made up of a loss by bottom friction, a loss by spilling of the wave and a loss by internal friction (very small). The correctness of this assumption can only be proved by experiments. Calculations give:

$$x = L_o \sqrt{2\pi y} \left[2\left(\frac{2\pi y}{L_o}\right) + \frac{1}{3}\left(\frac{2\pi y}{L_o}\right)^2 + \frac{43}{180}\left(\frac{2\pi y}{L_o}\right)^3 ----- \right]$$

where y is the water depth and L_o the deep water wave length. The series is convergent for $y < L_o/8$, i.e., for storm waves on the Danish West coast out to depths of about 40 feet where $L_o \sim 300$ feet. Since $y \ll L_o$, the equation may be reduced to

$$y^{3/2} = px \quad \text{where p is a constant.} \tag{1}$$

(II). If it now is assumed that the loss of energy is due only to bottom friction and that this loss per unit area, e_t, is constant, then

$\tau = k\rho\; u_{ave}^2$ where from Bagnold(1), $K = \text{constant} \cdot \left(\frac{a}{R}\right)^{3/4}$

a is the length of the ripple marks and R, the half amplitude of the oscillating water motion at the bottom ($R \geq a$).

Calculations then give:

$$y^{3/2} = p \frac{x}{T^{2/3}} \qquad (y < \text{about } \frac{L_o}{8}) \qquad \text{(II)}$$

This profile is similar to the one above. Certainly the profile depends on the wave period T but as the profile mainly is shaped by storm waves and as the variation in T for these is small, the profile in reality will be the same as that given by (I).

III. If $\frac{dE_1}{dt}$ and e_t are both assumed constant then:

$$y^2 = \frac{px}{T^{2/3}} \qquad (y < \text{about } \frac{L_o}{8}) \qquad \text{(III)}$$

If the loss of energy is mainly due to friction loss at the bottom, the assumption $\frac{dE_1}{dx}$ equals constant seems to be most logical. If the energy loss is primarily due to some other circumstance, especially to spilling of the wave, the assumption $\frac{dE_1}{dt}$ equals constant may be more logical. So there is more reason to expect to find the profile $y^{3/2} = px$ nearest to the shore line but outside the bar - and the profile $y^2 = px$ more distant from the shore line; this is the case, as investigations with beach profiles mentioned below show that the profile is flatter than $y^{3/2} = px$ at greater depths but follows this equation in shallow water (see Figure 22).

These calculations give the slope of the profile but naturally it cannot be maintained that the profile is "computed".

Keulegan and Krumbein (14) give a calcuation of a beach profile using the solitary wave theories of Boussinesq and Russell, and tests concerning the energy loss in a solitary wave. They found the equation:

$$y^{7/4} = \frac{x}{4.86} \sqrt[4]{\frac{\nu^2}{9}}$$

where y is the water depth; x, the distance from shore line; ν , the Kinematic viscosity; and g, the acceleration of gravity. The assumptions made in the derivation, however, seem to differ from actual conditions.

3.73 Use of the equilibrium profile in determining the future shore line recession. These calculations have been carried out under the assumption that future conditions of the shore regimen will be similar to these today. In addition it is assumed that the 0 to 30-foot area in the equilibrium profile is supplied with material eroded from the beach, and that at the same time some material is lost to bottom areas outside the 30-foot depth contour. The ratio between the quantity of material supplied to the 0 to x-foot area and the 0 to y-foot area is assumed to be x/y even though with our present state of knowledge we do not know to what degree the littoral drift outside, say, the 30-foot depth contour affects the littoral drift inside that contour. However it is a fact that the deposits which took place in the 20 to 30-foot area in 1934-1938 also occurred in 45 to 50-foot depths. From Table 6 it may also be seen that the landward acceleration of the 30-foot depth contour is mostly negative, (except for the areas near the channel). This possibly indicates that the equilibrium profile stretches farther seaward than the 30-foot depth contour (which is about 1/2 mile from the shore line).

The calculations of the shore line recession on the Northern Barrier were made up to 1916 because the increase in steepness after 1916 was moderate. For the same reason on the Southern Barrier the calculations were made after 1909.

As a consequence of the deposits in 1934-1938 (see section 3.4) the following calculations were carried through with both 1934 and 1938 as terminal dates. They were made for the whole of each of the Barriers as well as for the smaller stretches. L5-L8 on the Northern and L31-L34 on the Southern Barrier, these latter areas being used partly because they include the steepest profiles and partly because soundings were made there in 1942 and 1950, although it was not possible to make soundings over the entire Barrier area due to mine fields.

The calculations were made as described below: If the equilibrium profile $y_1^{3/2} = px_1$ is moved landward an increment, dx feet, the quantity eroded M_1 equals $y_1 dx_1$. With a depth of 30 feet and a crest elevation of the eroded portion of beach of 13 feet, the quantity eroded is 30 + 13 or 43 dx, cubic feet per foot of shore line. From the actual profile M_2 cubic feet are eroded with a corresponding shore line recession of dx_2 feet, which as a rule is less than dx_1 because of the effect of the groins in steepening the beach profiles. The theoretical and the actual eroded quantities were then made equal, as even detailed tables give no reliable basis for assuming a decrease in the quantity eroded within the periods considered, this being especially true for the Southern Barrier. Accordingly for the 30-foot depth:

$$43\, dx_1 = M_2 + 13\, dx_2$$

and

$$dx_1 = \frac{M_2 + 13\, dx_2}{43}$$

These calculations are also justified in the discussion of section 3.6 of a relation between shore line recession, bottom scour and movement of the 30-foot depth contour. The data used came from Tables 2, 21 and even more deatiled tables.

Table 27 shows the results of these calculations for different periods. The length of the periods considered is between 13 and 29 years as it is necessary to consider terms of several years to get reliable values for the quantity eroded (see section 3.4 & Tables 18 and 19). It is impossible to state the error of the calculated eroded quantities in general, but for the whole Barrier areas it has dimensions as indicated in Table 19, i.e. about 10 percent. For the areas L5-L8 and L31-L34, the average error is about 20 percent. The table also shows the result of the calculations for L5-L8 on the Northern Barrier for 1927, 1934, 1942 and for L31-L34 on the Southern Barrier for 1927, 1934, 1950. In addition two cases are shown for calculations based on the scour of small areas far from the shore. The inaccuracy of the deep water investigations, however, is considerable, and these results, therefore, are not very reliable. Table 28 shows the actual shore line recession after 1938.

TABLE 27 - FUTURE ANNUAL SHORE LINE RECESSION (FEET)

Basis of Calculation	Northern Barrier	L5 - L8
1916-1934	11.5	12
1916-1938	10	10
1921-1934	11.5	7.5
1921-1938	9	4.5
1927-1942		8.5
1934-1942		7
		16*
		10*
	Southern Barrier	**L31 - L34**
1909-1934	8.0	8
1909-1938	6.0	6
1916-1934	10.5	11
1916-1938	7.5	8
1927-1950		8.5
1934-1950		9
1936-1950		8 to 13**

* Calculated on basis of scour in 40 to 50-foot depth
** Calculated on basis of scour in 50 to 70-foot depth

TABLE 28 - FUTURE ANNUAL SHORE LINE RECESSION (FEET)

Periods	Northern Barrier NB	Northern Barrier L5 - L 8	Southern Barrier SB	Southern Barrier L31 - L33*
		Actual		
1938 - 1945	11.5	8.5	7	12
1938 - 1948	11.5	10	6	9
		Theoretical		
	11.5		8.5	

* No measurements at L34

3.74 Actual and theoretical eroded quantities. According to section 3.73 the quantity eroded out to a depth of y_1 feet associated with a shore line recession dx_1 feet is $(y_1 + 13)dx_1$ cubic feet per foot of the shore line.

With a shore line recession of 11.5 feet on the Northern Barrier and 8.5 feet on the Southern Barrier the quantities eroded annually per yard of the coast out to the 30-foot depth are about 55 and 41 cubic yards respectively.

Table 29 shows a comparison between the actual and theoretical quantities eroded for different periods on both Barriers. Regarding the error in the computation of the eroded quantities, see section 3.4, Tables 18 and 19.

TABLE 29 - ACTUAL AND THEORETICAL QUANTITIES ERODED ANNUALLY (CU.YDS/YD)

	Northern Barrier			Southern Barrier		
Periods	Eroded from the 0 to 30 ft. area	Eroded from the beach	Total	Eroded from the 0 to 30 ft. area	Eroded from the beach	Total
1921-1924	44	10	54	53	5	58
1921-1938	33	10	43	31	6	37
1927-1934	56	6	62	53	2	55
1927-1938	36	6	42	20	5	25
Theoretical	38	17	55	28	13	41

From this table it may be seen that there is a good agreement for the Northern Barrier, but for the Southern Barrier the actual quantities fluctuate around the theoretical. Table 30 shows a comparison between the actual and theoretical quantities eroded in the stretches L5 - L8 and L31 - L34 based on the most recent soundings.

TABLE 30 - ACTUAL AND THEORETICAL QUANTITIES ERODED ANNUALLY (CU.YDS/YD)

	(L5-L8) Northern Barrier			(L31-L34) Southern Barrier		
Periods	Eroded from the 0 to 30 ft. Area	Eroded from the beach	Total	Eroded from the 0 to 30 ft. area	Eroded from the beach	Total
1916-1942	38	12	50			
1909-1950				30	8	38
1921-1942	24	12	36			
1921-1950				32	10	42
1927-1942	32	8	40			
1927-1950				32	10	42
1934-1942	23	14	37			
1934-1950				29	13	42
Theoretical	38	17	55	28	13	41

Between L5 and L8 the agreement is excellent for 1916-1942, but not as good for the later periods, this possibly being due to the shortness of the periods considered as compared to the accuracy of the calculations. From 1938 to 1942 the quantity eroded was about 50 cubic yards per year. It is probable that an increase in shore line recession between L5 and L8 can be expected. It is therefore, interesting to see that the recession was 12 feet in 1945-1948 but 8.5 feet in 1938-1945.

Between L31 and L34 the agreement is excellent when the shore line recession was increased to 8.5 feet (13 cubic yards eroded from the beach) while, at the same time, the erosion in the 0 to 30-foot area decreased correspondingly.

In 1938-1950 the quantity eroded was 50 cubic yards, probably a reaction from the deposition in 1934-1938.

3.75 Actual and theoretical bottom scour. The scour may be calculated from the equations of the equilibrium profile as shown below.

For the Northern Barrier, the equilibrium profile is $y^{3/2} = 0.09x$ and

$$dy = \frac{0.133\ dx}{x^{1/3}}.$$

The shore line recession, dx, is 11.5 feet. Then $dy = \frac{18.4}{x^{1/3}}$ inches, where x is in feet.

For the Southern Barrier, the equilibrium profile is $y^{3/2} = 0.082x$ and $dy = \frac{0.127\ dx}{x^{1/3}}$.

The shore line recession, dx, is 8.5 feet. Then $dy = \frac{12.8}{x^{1/3}}$ inches, where x is in feet.

The average bottom scour in a certain area is the quantity eroded over the width of the area, i.e. $\frac{y_a\ dx}{a}$, where y_a is the depth at a distance a from the shore line, which recedes dx feet a year. The average scour between the two depth contours y_a and y_b is $\left(\frac{y_a - y_b}{a - b}\right) dx$.

TABLE 31 - ACTUAL AND THEORETICAL SCOURS (INCHES)

	Northern Barrier			L5 - L8			
Period	0-20'	20-30'	0-30'	Period	0-20'	20-30'	0-30'
1897-1934	2.8	3.2	3.0	1921-1942	1.3	1.7	1.5
1897-1938	2.8	2.4	2.7				
1909-1934	1.9	3.2	2.6	1927-1942	2.1	2.3	2.1
1909-1938	2.2	2.1	2.2				
1921-1934	1.2	3.5	2.5	1924-1942	1.9	0.7	1.4
1921-1938	1.7	1.7	1.9				
1927-1934	1.8	4.7	3.2				
1927-1938	2.2	1.4	2.0				
Theoretical	2.8	1.7	2.3	Theoretical	2.8	1.7	2.3
	Southern Barrier			L31 - L34			
1897-1934	1.6	1.6	1.6	1927-1950	1.2	1.9	1.6
1897-1938	1.5	1.0	1.2				
1909-1934	1.4	1.7	1.5	1934-1950	2.1	0.9	1.5
1909-1938	1.3	0.8	1.1				
1921-1934	1.8	3.1	2.4				
1921-1938	1.4	1.3	1.4	1938-1950	1.8	3.0	2.4
1927-1934	1.3	3.9	2.6				
1927-1938	1.1	0.9	0.9				
Theoretical	1.8	1.1	1.5	Theoretical	1.8	1.1	1.5

Table 31 shows a comparison between the actual and theoretical scours on both Barriers. (see Table 13 - more detailed tables are omitted).

The comparison between the actual and theoretical scours usually shows values greater than the theoretical for scours calculated on the basis of 1934, and values smaller than the theoretical for scours calculated on the basis of 1938. This is especially true for the 0 to 20- and 0 to 30-foot areas, but the scours in the 20 to 30-foot area may in some periods be greater than the theoretical even when the calculations are carried out on the 1938 basis. A similar comparison may be made for the smaller areas, L5 - L8 on the Northern Barrier and L31 - L34 on the Southern Barrier, where the development is most advanced. Between L5 and L8 the measured scours are too small in the 0 to 20 and 0 to 30-foot areas and a little too large (1927 - 1942) or too small (1934-1942) in the 20 to 30-foot area. Between L 31 and L34 the scours in the 20 to 30 and 0 to 30-foot areas are smaller than the theoretical in 1934-1950 but greater than the theoretical in 1927-1950. The scours in 1938-1950 are also greater, possibly a reaction from the shoaling in 1934-1938 (see sections 3.4 and 3.54). From the above it appears as though the development of the 20 to 30-foot area still is not quite stable.

3.76 Actual and theoretical mean depths and steepness characteristics. Calculations on the basis of the equilibrium profile give:

$$\text{mean depth} = md = \frac{3}{5} y_a$$

$$\text{steepness characteristic} = stc = \frac{3\, y_a}{5a}$$

where y_a is the depth at a distance a from the shore line.

In Table 32 md and stc are computed for three different types of beach profiles corresponding to the equilibrium profiles on both Barriers. The profiles go through the two points 0,0 and 1815, 30 on the Northern Barrier the points 0, 0 and 1980, 30 on the Southern Barrier (see section 3.72).

TABLE 32 - MEAN DEPTHS AND STEEPNESS CHARACTERISTICS FOR DIFFERENT PROFILES

	Mean Depth (ft)		Steepness Characteristic x 10^3			
	Both Barriers		Northern Barrier		Southern Barrier	
Profile	0 - 20'	0 - 30'	0 - 20'	0 - 30'	0 - 20'	0 - 30'
straight line	10	15	8.2	8.2	7.5	7.5
$y^{3/2} = 0.09x$	12	18	12.0	9.8	11.0	9.0
$y^{3/2} = 0.82x$	13	20	16.3	10.9	15.0	10.0

Tables 33 and 34 show theoretical and actual values of md and stc for both Barriers, and for the areas L5-L8 on the Northern Barrier and L31 - L34 on the Southern Barrier. The errors (or standard deviations) involved were discussed in section 3.24, where they were all found to be small.

TABLE 33 - ACTUAL AND THEORETICAL MEAN DEPTHS (FEET)

Year	NORTHERN BARRIER				SOUTHERN BARRIER			
	Northern Barrier		L5 - L8		Southern Barrier		L31-L34	
	0 - 20'	0 - 30'	0 - 20'	0 - 30'	0 - 20'	0 - 30'	0 - 20'	0 - 30'
1934								
Actual	12.5	18	12	18	12	18	12	18.5
Theoretical	12	18	12	18	12	18	12	18
1938								
Actual	14	18.5	13.5	18	12.5	18.5	13	19.5
Theoretical	12	18	12	18	12	18	12	18
1942								
Actual			12	18				
Theoretical			12	18				
1950								
Actual							12	18
Theoretical							12	18

TABLE 34 - ACTUAL AND THEORETICAL STEEPNESS CHARACTERISTIC X 10^3

Year	NORTHERN BARRIER				SOUTHERN BARRIER			
	Northern Barrier		L5 - L8		Southern Barrier		L31 - L34	
	0 - 20'	0 - 30'	0 - 20'	0 - 30'	0 - 20'	0 - 30'	0 - 20'	0 - 30'
1934								
Actual	12.8	9.9	13.9	11.0	11.5	8.9	11.9	8.9
Theoretical	12.0	9.8	12.0	9.8	11.0	9.0	11.0	9.0
1938								
Actual	12.0	9.1	12.1	10.0	11.3	8.2	13.0	8.4
Theoretical	12.0	9.8	12.0	9.8	11.0	9.0	11.0	9.0
1942								
Actual			13.2	11.0				
Theoretical			12.0	9.8				
1950								
Actual							11.0	8.8
Theoretical							11.0	9.0

* * * * * * * * * * * * * * * * * * *

These Tables, and especially the data for 1942 and 1950 which show a comparison for the profiles with the most advanced development indicate that there is a fairly good agreement between the theoretical and measured values.

The above seems to show that it is possible to describe the development by means of an "equilibrium profile" which on the Northern Barrier will follow the equation $y^{3/2} = 0.09x$ and on the Southern Barrier the equation $y^{3/2} = 0.082x$. These equations are valid out to depths of 40 to 50 feet. It has been shown probable that the future annual shore line recessions under the conditions now existing will be about 11.5 and 8.5 feet on the Northern and Southern Barriers respectively.

4. CONCLUSIONS

The investigations described above show:

a. Development of the planform of the Barriers. The sea bottom outside the Lime Inlet Barriers is under continuous erosion. No soundings are available which show the depth at which erosion stops. A few profiles sounded in 1936, 1938 and 1950 indicate that erosion probably still takes place at depths of 60 to 70 feet.

The breach of the Barriers caused the coast to develope a "funnel". The influence area of the channel has enlarged to both sides and it appears to extend from about L 14A on the north to about L48 on the south or perhaps 1 to 2 miles farther southwards (Figure 3). The point of maximum shore line recession, as far as can be seen, has also moved away from the channel and is now at the unprotected areas at L1 on the north and L40 on the south. Considerations on the basis of the bottom scour and of "mass-curves" for the quantity eroded give the same results. Tt appears that the 0 to 20-foot area is influenced first and then the 20 to 30-foot area. As far as can be seen the enlargement of the influence area of the channel is greatest to the south (L37 - L48) where the entire 0 to 30-foot area is influenced, whereas to the north (L1 and northward) essentially only the 0 to 20-foot area is eroded. Perhaps the development described above can be generalized for the development of barrier seacoasts around inlets. The shore line recession has decreased greatly since the groins were built although the groins have not influenced the erosion beyond their distal ends, i.e., outside the 20-foot depth contour. The minimum shore line recession occurred in 1927-1934 on protected as well as unprotected reaches, and this perhaps was due to the relatively calm weather conditions in this period (see section 3.3). This again means that the minimum shore line recession in this period cannot be explained as a result of a maximum influence of the groins before the development accelerates landward. After 1934 the shore line recession appears to have increased on both barriers. This recession gradually became fairly similar along the entire length of the Barriers, so that the form of the coastline no longer changed appreciably.

The total movement of the shore line, as shown in Table 4, may be divided into three different types of movement; a) migrating waves; b) seasonal fluctuations; and c) movements of the entire beach profile because of erosion. The latter is smallest and, therefore, it is difficult to draw reliable conclusions about the erosion of the sea bottom on the basis of shore line recession.

From investigations of the speed and acceleration of the movement of the shore line and the 20 and 30-foot depth contours off the Barriers it appears that the shore line movement will accelerate somewhat landward, while the movement of the 20 and 30-foot depth contours will probably stabilize at a particular level, but at the same time be more irregular.

b. Development of the beach profile. During the development the mean depth has remained almost constant. Investigations of the development of the steepness show that all the coasts considered steepen. This development is most pronounced for the Barriers, which undoubtedly is a result of the flattening out of the beach profiles immediately after the breach of the channel.

The development of the steepness characteristic of the beach profile shows that the coast between L1 and L 14A has always been the steepest. The increase in steepness since 1921 was greatest on the Southern Barrier; but almost the same on the Northern Barrier as on the unprotected coast L38 - L47. The maximum steepness out to the 30-foot depth contour occurred everywhere in 1934 except for L58 - L61 where special conditions prevailed on the leeside of the groin at Bovbjaerg. The greatest absolute steepness north of the channel was on the profiles on the unprotected area at L4 (stc x 10^3 = 13.5). South of the channel, L31 in the protected area was the steepest (stc x 10^3 = 10.4), but about twenty beach profiles on the unprotected coast L4 - L 14A were as steep or steeper in different periods. The most recent soundings L5 - L8 on the Northern Barrier in 1942, show that L8 is steeper than ever before, while L5 - L7 have been steeper in different periods. On the Southern Barrier soundings L31 - L34 in 1950 show that L 33 is steeper than ever before but about twenty profiles on the Barriers have been steeper in different periods. L31, L32, and L34 have been steeper before 1950. The average steepness L5 - L8 was greatest in 1934-1942; L31 - L34 was steepest in 1934-1950. It appears that beach profiles "wriggle" onshore, being sometimes less steep and sometimes perhaps a little more steep than the profiles in 1934.

There seems to be a relation between increase in steepness and shore line recession. As the steepness increases the profile fluctuates more and more and, as mentioned above, material is "drawn out" from the beach for replacement of material eroded outside the 0 to 20-foot depth contour.

There seems to be a fairly good relation between the fluctuations in steepness and the wind (wave) conditions as periods with proportionately more and stronger storms coincide with periods of decreasing steepness. In this way it appears that the short period fluctuations shown by laboratory and field experiments (see section 3.3) can be extended to include long period fluctuations of the same kind which again means that the development of the beach profiles is very "sluggish".

c. Bottom scour. The bottom scour is greatest along the Northern Barrier, about 2 to 3 inches a year; on the Southern Barrier it is about 1.5 inches and 1 to 1.5 inches on the coasts L1-L14A and L37-L48; it is least south of L48, being 1/2 to 1 inch a year.

On the Barriers there seems to be a tendency toward stabilization of this process in the 0 to 30-foot area, but otherwise the scour seems to be irregular.

For all the coasts investigated several years' continuous scour of the 20 to 30-foot area was followed by a period where material from the bottom inside the 0 to 20-foot depth contour was "drawn out" into deeper water. On the Southern Barrier this development probably began about 1900 and somewhat later on the Northern Barrier which seems to indicate that the Southern Barrier is older in development than the Northern. However, this transverse movement frequently lags several years behind the scour in the 20 to 30-foot area.

There is a relation between shore line recession, bottom scour and width of the 0 to 30-foot area, and it is possible to calculate the latter when the two former are known (section 3.6).

d. Quantity eroded. On the Northern Barrier the annual quantity eroded between the shore line and the 30-foot depth contour shows a tendency to stabilize on about 400,000 cubic yards, corresponding to an average annual shore line recession of about 11.5 feet. On the Southern Barrier the corresponding numbers are about 300,000 cubic yards per year and about 8.5 feet. It has been impossible to derive a relationship between the quantity eroded and the wind conditions (wave conditions).

e. Equilibrium profiles. It is possible to describe the development of the beach profile and the erosion by means of an equilibrium profile; that is, a stable profile with maximum steepness. This profile will follow the equation $y^{3/2} = 0.09\ x$ on the Northern Barrier and $y^{3/2} = 0.082\ x$ on the Southern Barrier (where y is the water depth in feet at a distance x from the shore line). Under the assumption that the future annual shore line recession on the Northern and Southern Barriers respectively will be 11.5 and 8.5 feet a year, there was a relatively good agreement between actual and theoretical erosion, mean depths and steepness characteristics.

PART II - MISSION BAY, CALIFORNIA AND COMPARISON WITH DANISH DATA

5. SHAPE OF THE BEACH PROFILES

5.1 DESCRIPTION. The Mission Bay area is shown on Figure 22. As far as can be determined, the La Jolla and Point Loma headlands prevent any appreciable littoral drift into and out of the area. These are slightly eroded and supply to the bay area small amounts of material, mainly cobbles from La Jolla and possibly some finer material from Point Loma. However, much of the material eroded from Point Loma is of silt size which is carried to deep water.

There is a seasonal migration of sand between the offshore and shallow water regions, but the predominant feature of the beach area under consideration is its close approach to a conservative system(24).

5.2 SHAPE OF PROFILES. Surveys of Mission Bay Channel and statistical studies of currents in the surf zone are mentioned in (27). The shape of the actual beach profile is dependent upon such factors as the wave characteristics and the change in these characteristics as the wave moves into shallow water, particularly as this is affected by the difference in the direction of wave propagation. In addition it must be assumed that the beach profile is a function of grain size and its distribution as well as the specific weight of the grains. The coastal currents, and more especially the longshore current, may also play an important role.

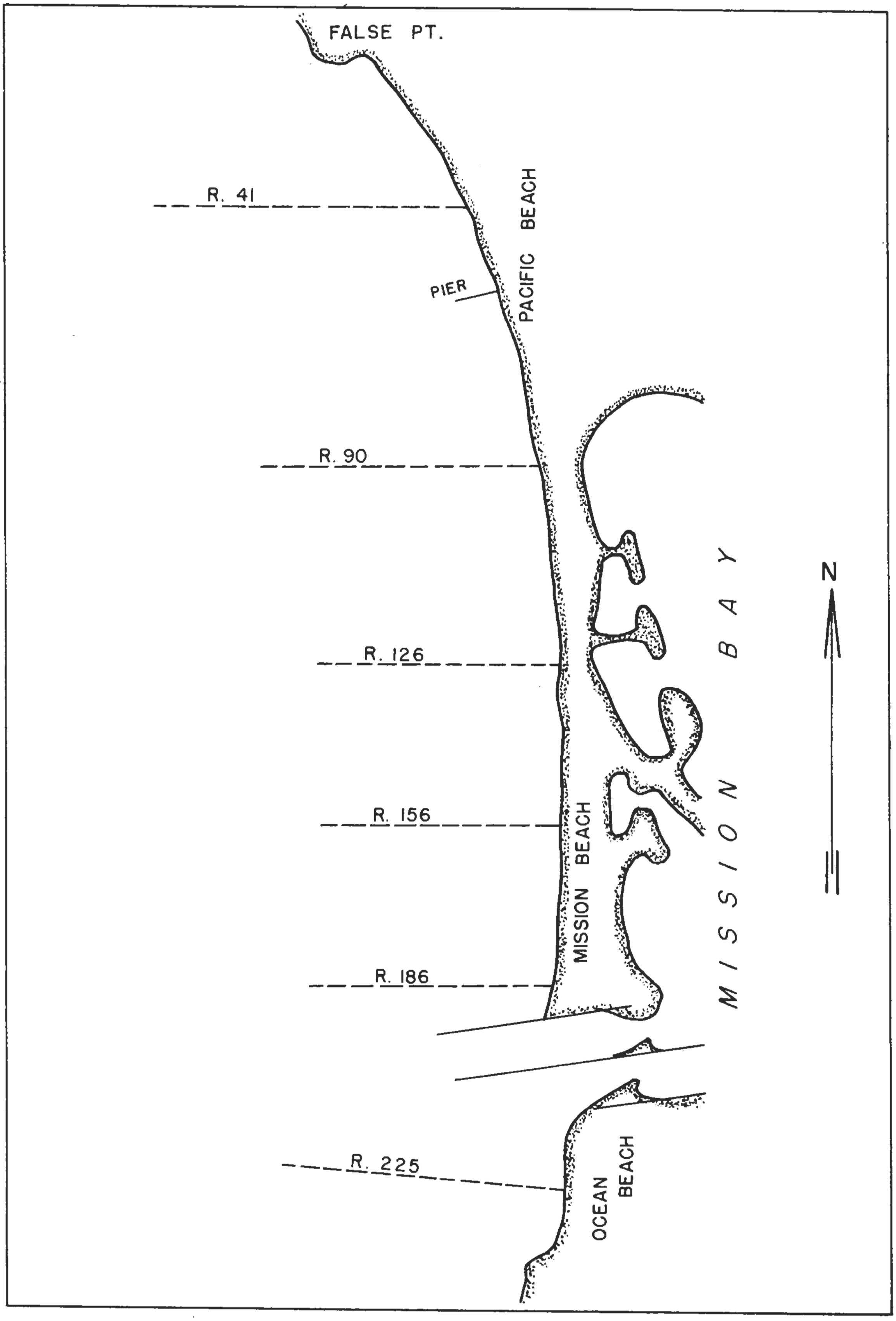
Ultimately the shape depends on the initial conditions. On extremely steep coasts there may be a single bar or even none. It appears as though the number of bars is dependent upon the magnitude of littoral drift, as it increases so does the possibility of bar formation. However, it does not follow that a profile with, say three bars, necessarily carries more material than one with a single bar, for many other factors also influence the quantity of littoral drift.

Laboratory experiments have shown the existence of "equilibrium profiles", which, are defined in section 3.71 as profiles which maintain their form. Under actual conditions the equilibrium profile must be defined as a statistical average profile which maintains its form despite minor fluctuations in the time interval considered.

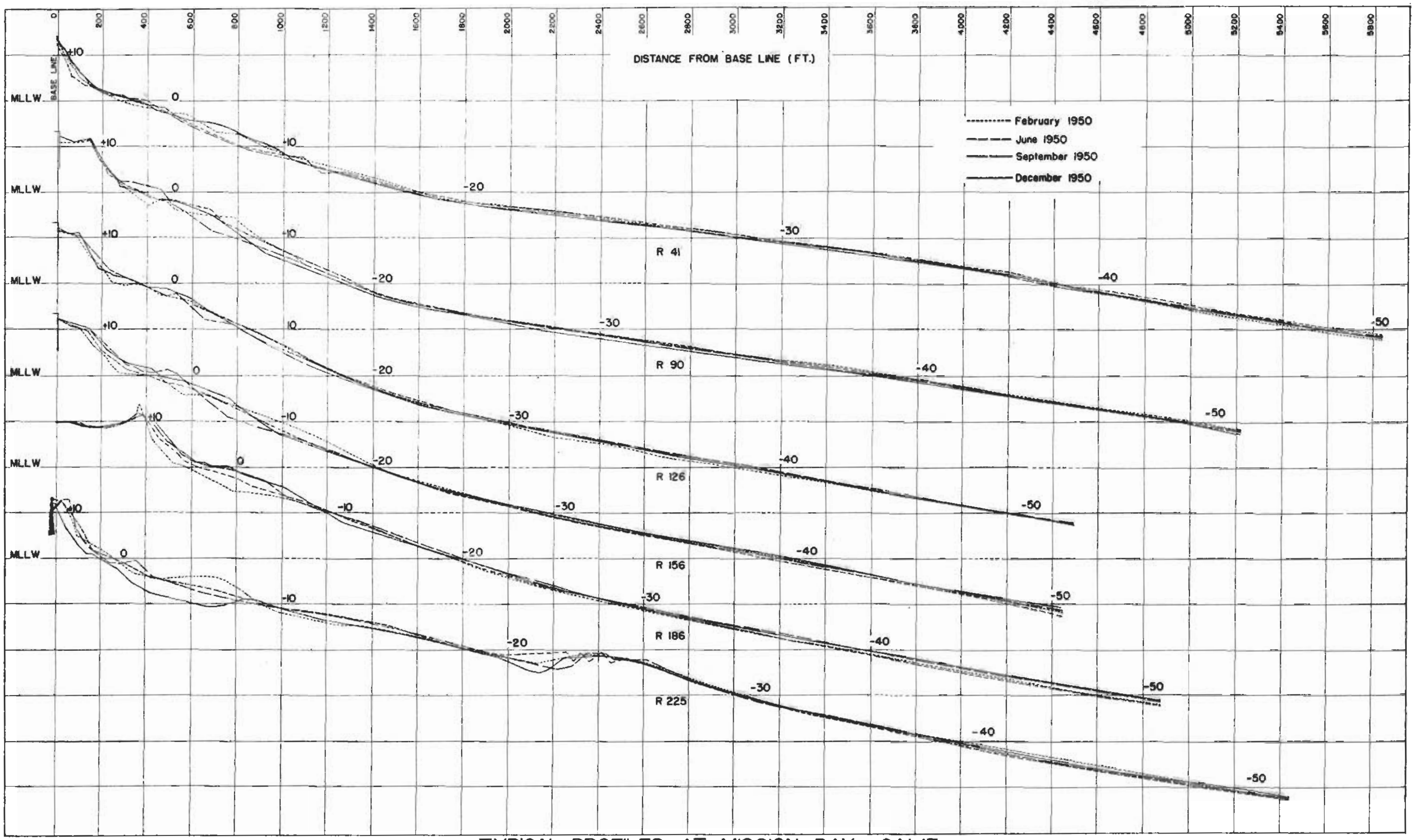
Considerations based upon the development of beach profiles on the Danish North Sea coast (which is composed of sand having a grain size between 0.2 and 0.3 mm., see Figure 2) seem to show that one must distinguish between three types of beach profiles:

over-nourished, sufficiently-nourished and under-nourished.

The over-nourished profiles are fed with more material than the waves can shape into a beach profile. Very often, therefore, they are very irregular and appear as shoals.



MISSION BAY, CALIF. – PROFILE LOCATIONS
FIGURE 22



TYPICAL PROFILES AT MISSION BAY, CALIF.
FIGURE 23

There are two different types of sufficiently-nourished profiles. In one the profiles are not fed with more material than the waves can shape into a profile, (which has the "equilibrium form"). In the other the loss of material equals the supply of material, and the profile has still the same equilibrium form.

The under-nourished beach profiles are eroded and the coastline recedes. The under-nourished beach profiles will always maintain an equilibrium form but this form may change from one locality to another.

From the above it can be seen that advance of a coastline may take place with or without an equilibrium profile while recession of a coastline can probably only take place with equilibrium profiles which have maximum steepness corresponding to the quantity of littoral drift. An actual equilibrium profile, therefore, should be defined as "a stable profile with maximum steepness", apart from seasonal fluctuations.

As the Mission Bay beach seems to be a sort of "pocket beach" with fairly stable conditions, there is reason to believe that the steep beach profiles in this area are sufficiently-nourished, or perhaps slightly under-nourished, equilibrium profiles.

An investigation of the grain size distribution shows that the median grain size decreases seaward from the shore (Table 35). This is a sign of profile maturity.

TABLE 35 - DISTRIBUTION OF GRAIN SIZE, MISSION BAY

(Median diameters in mm.)

Depth (feet)	Range 136	Range 170
10	0.144	0.143
20	0.110	0.136
30	0.094	-----
40	0.089	0.107
50	0.080	0.097

There are therefore several reasons to believe that the steep profiles at the center of the Mission Bay beaches are equilibrium profiles.

Investigations in this respect were carried out with profiles 90, 126, and 156, for investigations of "seasonal fluctuations in general" the five profiles 41, 90, 126, 156 and 186 were used, (see Figures 22 and 23). Profiles sounded in February, April and December 1950 were considered as "winter profiles" while those taken in June and September 1950 were considered as "summer profiles", based on observed wave characteristics. Naturally these terminologies cannot be identified as "bar profiles" and "beach ridge profiles" when compared with laboratory experiments with beach profiles, (see Table 38).

TABLE 36 - AVERAGE BEACH PROFILE DATA FOR FIVE PROFILES

Range	Area 0-20' (sq. yds.)	Width 0-20' (yds.)	md (ft.)	stc x 10^3	Area 0-30' (sq. yds.)	Width 0-30' (Yds.)	md (ft.)	stc x 10^3
				FEBRUARY				
41	1450	440	10	7.6	5700	950	18	6.3
90	950	325	9	9.0	3650	645	17	8.8
126	950	325	9	9.0	2800	545	15.5	9.3
156	1050	340	9.5	9.1	3050	580	16	9.2
186	1300	390	10	8.6	3250	625	15.5	8.3
Sum R90-R156	2950	990	27.5	27.1	9500	1770	48.5	27.3
Sum R41-R186	5700	1820	47.5	43.3	18450	3345	82.0	41.9
Avg R90-R156	980	330	9.2	9.0	3170	590	16.2	9.1
Avg R41-R186	1140	365	9.5	8.7	3690	670	16.4	8.4
				APRIL				
41	1500	450	10	7.4	5500	960	17	6.0
90	950	310	9	9.9	3300	590	17	9.6
126	1000	300	10	11.1	2800	530	16	10.1
156	1050	320	10	10.3	3000	570	16	9.3
186	1250	385	9.5	8.4	3300	625	16	8.5
Sum R90-R156	3000	930	29	31.3	9100	1690	49	29.0
Sum R41-R186	5750	1765	48.5	47.1	17900	3275	82	43.5
Avg R90-R156	1000	310	9.7	10.4	3035	565	16.3	9.7
Avg R41-R186	1150	355	9.7	9.4	3580	655	16.4	8.7
				JUNE				
41	1550	420	11.	8.8	5750	960	18	6.2
90	1000	320	9.5	9.8	3450	640	16	8.4
126	950	310	9	9.9	2700	525	15.5	9.8
156	1050	325	9.5	10.0	3100	565	16.5	9.7
186	1150	375	9	8.2	3300	625	16	8.5
Sum R90-R156	3000	955	28.0	29.7	9250	1730	48.0	27.9
Sum R41-R186	5700	1750	48.0	46.7	18300	3315	82.0	42.6
Avg R90-R156	1000	320	9.3	9.9	3085	575	16.0	9.3
Avg R41-R186	1140	350	9.6	9.3	3660	665	16.4	8.5
				SEPTEMBER				
41	1500	390	11.5	9.9	5700	885	19.5	7.4
90	1050	280	11	13.4	3600	585	18.5	10.5
126	950	300	9.5	10.6	2850	525	16.5	10.3
156	1150	285	12	14.2	3250	525	18.5	11.8
186	1200	360	10	9.3	3200	600	16	8.9
Sum R90-R156	3150	865	32.5	38.2	9700	1635	53.5	32.6
Sum R41-R186	5850	1615	54.0	57.4	18600	3120	89.0	48.9
Avg R90-R156	1050	290	10.8	12.7	3235	545	17.8	10.8
Avg R41-R186	1170	325	10.8	11.5	3720	625	17.8	9.8
				DECEMBER				
41	1400	410	10.5	8.3	5350	900	18	6.6
90	950	300	9.5	10.6	3250	600	16	9.0
126	900	285	9.5	11.1	2850	520	16.5	10.6
156	1000	310	9.5	10.4	3200	570	17	9.9
186	1100	355	9.5	8.7	3400	600	17	9.5
Sum R90-R156	2850	895	28.5	32.1	9300	1690	49.5	29.5
Sum R41-R186	5350	1660	48.5	49.1	18050	3190	84.5	45.6
Avg R90-R156	950	300	9.5	10.7	3100	565	16.5	9.8
Avg R41-R186	1070	330	9.7	9.8	3610	640	16.9	9.1

TABLE 37 - COMPARISON OF ACTUAL AND THEORETICAL EQUILIBRIUM PROFILES

	Area 0-20' (sq.yds)	Width 0-20' (yards)	md (ft.)	stc x 10^3	Area 0-30' (sq. yds.)	Width 0-30' (yards)	md (ft.)	stc x 10^3
			MISSION BAY					
Winter-Actual	980	315	9.5	10.0	3100	575	16.3	9.6
Theoretical	1240	310	12.0	12.9	3420	570	18.0	10.5
Summer-Actual	1025	305	10.5	11.0	3160	560	17.0	10.1
Theoretical	1200	300	12.0	13.3	3300	550	18.0	10.9
	Steepest average equilibrium profile on the Danish North Sea Coast							
NB, 1934	1400	330	12.7	12.8	3650	605	18.1	9.9
NB, L5-L8,1942	1250	305	12.2	13.2	3150	535	17.8	11.0
Theoretical	1320	330	12.0	12.0	3650	605	18.0	9.8

Table 36 shows the water area above the five beach profiles out to -20 and -30 feet. In addition, the corresponding widths of the 0 to 20 and 0 to 30-foot areas, the values of mean depth (md) and steepness characteristics (stc) are given. Naturally the average values of md and stc cannot be calculated directly from the average areas and widths.

For investigations of the profile shape, the average winter profile may be approximated by the equation $y^{3/2} = 0.096x$ where x is the distance from the shore line and y is the depth (both in feet - see section 3.72); the summer profile may be approximated by $y^{3/2} = 0.1x$. The corresponding "theoretical" values of area, width, md, and stc have been calculated and are tabulated in Table 37, It may be seen that the agreement in widths is good, but the theoretical values of md and stc are always a little larger than the observed, especially for the 0 to 20-foot area where the actual profile tends to follow a straight line (Table 32).

In Table 37 comparison is also made with an average profile on the Northern Lime Inlet Barrier, the calculations being on the basis of the sounding of 15 profiles in the summer of 1934(section 3.76, Tables 33 and 34). Also shown are the data from the four extremely steep profiles, L5 - L8, on the Northern Barrier (see section 3.76, Tables 33 and 34) and the theoretical average profile for the Northern Barrier, $y^{3/2} = 0.09x$. It may be seen that there is not too much difference between the Mission Bay summer profile, and the steep average profile, L5 - L8, in Denmark (also sounded in the summer season). In the main the difference is that the Mission Bay profiles are straighter between the shore line and the 20-foot depth contour, while the Danish profiles may be provided with a bar -- or more a rudiment of a bar -- and a fairly deep trough. (Figure 21).

In Figure 24 the Mission Bay average summer and winter profiles (indicated by black points) are compared with the "theoretical profiles". The excellent agreement should not be taken too literally.

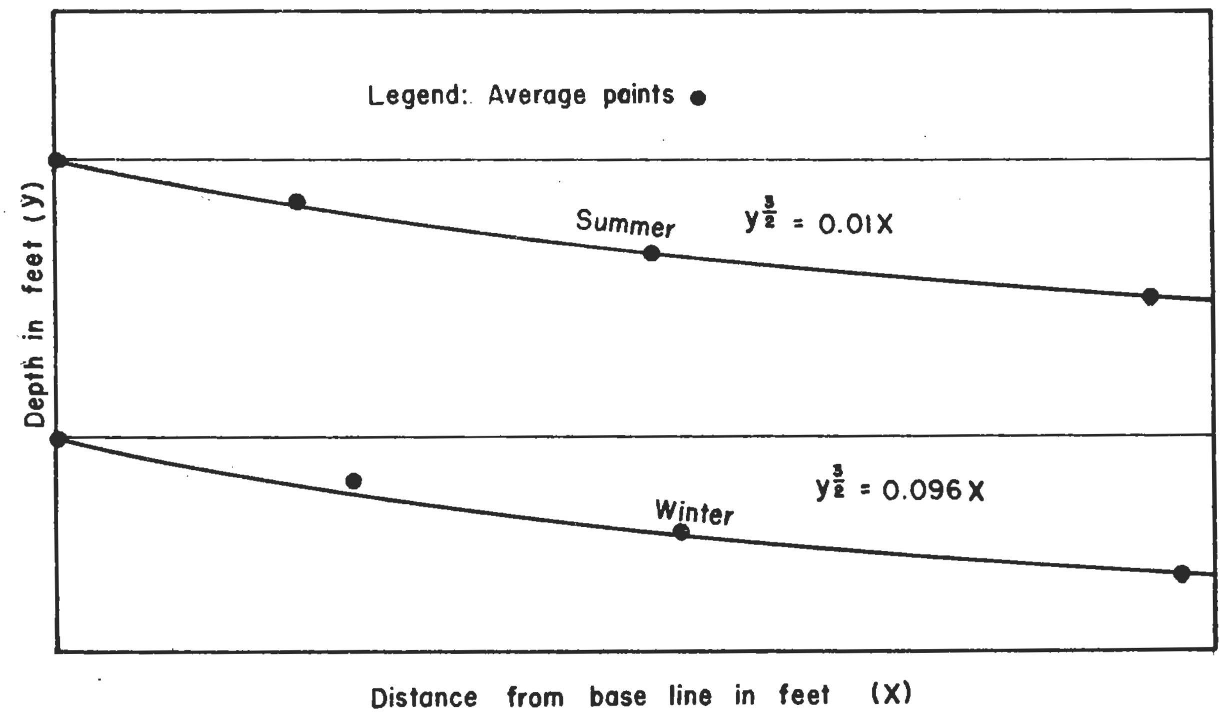


FIGURE 24 - MISSION BAY AVERAGE AND THEORETICAL PROFILES

5.3 SEASONAL FLUCTUATIONS OF THE BEACH PROFILES. Tables 38 shows the values of md and stc for a d/L_o ratio of 0 to 0.06 and H_o/L_o ratios of 0.018 and 0.043, obtained from laboratory experiments with beach profiles in Copenhagen.

TABLE 38 - MD AND STC FOR DIFFERENT EQUILIBRIUM PROFILES IN THE LABORATORY (d/L_o = 0 to 0.06)

Season	Type of Profile	H_o/L_o	md	stc x 10^3
Summer	Beach ridge	0.018	0.05	75
Winter	Bar	0.043	0.03	50

From Table 38 it may be seen that md as well as stc is greater for swells than for the steeper storm waves.

Table 39 is derived from the data in Table 36 and shows a comparison between the average values for the five winter profiles (sounded in February, April and December 1950) and those for the summer profiles (sounded in June, and September, 1950). Although being designated as summer and winter profiles respectively, it may be noted that the June and December profiles are actually in more of a transitional stage - the one being a little flatter and the other a little steeper than ought be expected in a true summer or winter profile.

TABLE 39 - COMPARISON OF WINTER AND SUMMER PROFILES, MISSION BAY

Season	0 to 20-ft. Area sq. yds.	Width yards	md ft.	stc x 10^3	0 to 30-ft. Area sq. yds	Width yards	md ft.	stc x 10^3
Winter	1120	350	9.6	9.3	3625	655	16.6	8.7
Summer	1155	335	10.2	10.4	3690	645	17.1	9.2

From Table 39 it may be seen that the areas are only slightly different. The width, md and stc are all less for the winter profile than for the summer profile, just as in the laboratory. The three profiles in Table 37 have fluctuated in the same way as the five profiles considered.

TABLE 40 - COMPARISON OF WINTER AND SUMMER PROFILES. DANISH NORTH SEA COAST AT BOVBJAERG

Season	0 to 20-ft. Area (sq. yds.)	Width (yards)	md (ft.)	stc x 10^3	0 to 30-ft. Area (sq. yds)	Width (yards)	md (ft.)	stc x 10^3
3/28/52 Winter	2390	525	13.6	8.7	5850	795	22.1	9.3
7/15/52 Summer	2370	495	14.4	9.7	6000	785	22.9	9.7

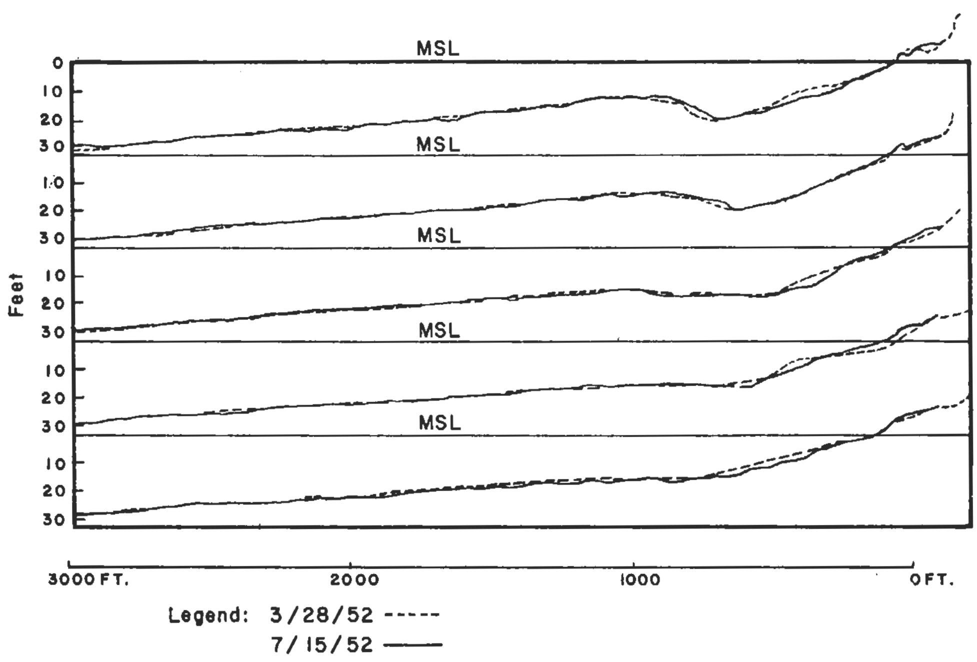


FIGURE 25- FLUCTUATIONS OF PROFILES SOUTH OF BOVDJAERG, DENMARK

Figure 25 and Table 40 show similar results of two soundings carried out March and July 1952 at Bovbjaerg south of the Lime Inlet Barriers (see Figures 2 and 3) in five lines of soundings spaced 300 feet apart. The bottom is composed of sand . From Figure 25 it may be seen that the March profiles are winter profiles and the July profiles summer profiles with "beach ridges". It appears that the bar has migrated shoreward. It is difficult to tell how far from the shore line seasonal fluctuations take place (30 feet). There is no equilibrium between the quantities deposited on the beach and those eroded from the sea bottom (the difference for the period considered being about 40,000 cubic yards eroded from winter to summer for the reach of about 1,300 feet). However, it appears as though material migrates on the bottom along the shore in "waves" or humps. This problem will be further discussed in a paper at the Fifth Conference on Coastal Engineering.

From Table 40 it may be seen that the areas differ but little. The width, md and stc are all less for the winter profile than for the summer profile. A comparison with the results from Mission Bay, Table 39 shows full agreement in fluctuations. There is, moreover, only a minor difference in stc to 30 feet between Mission Bay and Bovbjaerg. Yet the profiles at Bovbjaerg are eroded and situated on a coast with a strong littoral drift. The profiles are flatter than the steep profiles from the Lime Inlet Barriers which especially may be seen in the widths which are always greater for both the Lime Inlet Barriers and Mission Bay (see Table 37).

5.4 RELATION BETWEEN DEVELOPMENT OF MD - STC AND OBSERVED WAVES. An attempt is described below to find a relation between the development of the md-stc and the observed waves. It is assumed that all waves have a direction perpendicular to the shore line, which naturally is incorrect, although because of the shape of the bay it is a reasonable approximation.

Table 41, prepared by Forrest(24), shows the number of 6-hour periods of occurrence of waves of stated height for each month.

In Table 42 these observations are summarized, the observations being divided into groups with wave heights less than and greater than 4 feet. The wave height varied from 0.5 to 7 feet while the wave period very seldom was less than 10 or greater than 16 seconds; that is the wave length varied between about 500 and 1,300 feet. The average wave period and the average wave steepness are also shown. Comparison may be made with the values of stc in Table 39.

TABLE 41 - NUMBER OF 6-HOUR PERIODS OF OCCURRENCE OF WAVES

Wave height (feet)

1950 Months	0 - 1	1.0-1.9	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9
January	1	57	53	12		1	
February	3	60	47	2			
March	4	82	34	4			
April	28	74	8	7	2	1	
May	6	101	16	1			
June	30	82	8				
July	30	94					
August	35	87	2				
September	33	64	19	4			
October	24	60	25	8	2	3	2
November	43	56	19	1	1		
December	12	46	40	18	6	2	

TABLE 42 - COMPARISON OF WAVE DATA AND STC

	Wave obs.per mo.		Avg. period		Avg. wave lengths		stc x 10^3	stc x 10^3
Season	Height <4 ft.	Height 4-7 ft.	Height <4 ft.	Height 4-7 ft.	Height <4 ft.	Height 4-7 ft.	out to -20 feet	out to -30 feet
Summer	122.4	0.0	14		1000		10.4	9.2
Winter	118.3	2.9	13	15	870	1150	9.3	8.7

The average steepness ratio for waves less than 4 feet in the "winter season" (October to May) is about 0.002; in the "summer season" (May - October) it is about 0.0014. The average steepness ratio for waves from 4 to 7 feet in the winter season is 0.0045. No reliable conclusions in detail can be drawn on the basis of average values even though the above seems to indicate that a few intervals in January and April with a wave steepness ratio of 0.008 to 0.009 may have played a role in decreasing the steepness of the profiles. Besides waves from two different directions sometimes occurred at the same time so that the steepness ratio of the waves was increased above 0.01 by interference. Moreover the decrease in stc from summer to winter may not necessarily be caused by the steepening of the waves alone but also by the absolute wave heights. Great accumulations took place in the area in the winter season, and this may be a reason for believing that material from outside the 30-foot depth contours was pushed landward in the winter season (see sections 3.3 and 3.54).

Naturally there is no reason to believe that the profile steepness should change from increasing to decreasing steepness at the same H_o/L_o value as in the laboratory.

As previously stated, it is very difficult to make a comparison between the development of the steepness of the beach profile and average values of the wave characteristics. The beach profile depends not only on the steepness ratio H_o/L_o but also, naturally, on the absolute values of the wave characteristics. In particular, a high wave with a high H_o/L_o value may cause a large change in the profile in a short period of time. Observations with the same H_o/L_o, therefore do not have the same weight.

In the following a comparison is made between beach profile characteristics within a very limited section of the coast, and the wave characteristics. The time intervals were chosen to give as much change from one period to another as possible. Four beach profiles in the so-called "test section", where soundings were made almost every week from May 12 to September 8, 1950, were used. The particular profiles investigated were Nos. 126, 128, 130 and 132, (see Figures 22 and 23). Their spacing was 200 feet.

Table 43 shows the areas of the beach profile out to -20 and -30 feet and the corresponding widths, mean depths and steepness characteristics. The average wave heights, wave periods, and steepness ratios are also shown. From this table it may be seen that the 0 to 20-foot area decreased in size from May 18 to August 18 primarily because of the decrease in width. The increase from August 18 to September 1 may be due to a deepening of the area caused by the swells without a corresponding decrease in width. The 0 to 30-foot area shows the same fluctuations but in a relatively minor degree. The width of the 0 to 20-foot area decreased continuously while there was a slight increase in the width of the 0 to 30-foot area between May 18 and June 9, due to a smaller deposition of material at a depth of about 30 feet. The low swells probably are responsible for the decrease in width but it is questionable whether the increase of the 0 to 30-foot area between May 18 and June 9 may also be attributed to the swells. The mean depth up to -30 feet decreased somewhat May 18 - June 9 and then increased, but has a tendency to increase for the 0 to 30-foot and 0 to 30-foot areas.

The steepness characteristic for the 0 to 20 foot area has increased continuously. In the 0 to 30-foot area there is a decrease in size during the same period as the increase in width (May 18 - June 9) caused by deposition at about the 30-foot depth.

6. CONCLUSIONS

The steep profiles in the middle of the Mission Bay area seem to follow the equation $y^{3/2} = px$ with p being about 0.01 to a reasonable degree of accuracy.

The width of the 0 to 20 and 0 to 30-foot areas decreases from winter to summer season, mean depths and steepness characteristics increase - both in California and in Denmark.

TABLE 43 - AVERAGE BEACH PROFILE CHARACTERISTIC

(Mission Bay, California; profiles 126, 128, 130 132)

Date 1950	Area 0 to 20 ft. (sq.ft.)	Area 0 to 30 ft. (sq.ft.)	Width 0 to 20 ft. (ft.)	Width 0 to 30 ft. (ft.)	md 0 to 20 ft. (ft.)	md 0 to 30 ft. (ft.)	stc x 10^3 0 to 20 feet	stc x 10^3 0 to 30 feet	Ave. wave height H_o feet	Ave. wave period seconds	Ave. H_o/L_o
May 18	9210	26100	950	1600	9.70	16.29	10.21	10.18	0.97	12.62	0.0012
Jun 9	9200	25800	940	1630	9.79	15.82	10.46	9.72	1.04	11.76	0.0015
Jun 23	9000	25500	925	1610	9.71	15.82	10.58	9.83	0.86	13.86	0.0009
Aug 18	8680	25400	870	1545	9.99	16.42	11.48	10.63	0.93	13.48	0.0010
Sep 1	8900	26100	845	1515	10.52	17.22	12.47	11.38			

It appears that waves with wave steepness of 0.001 to 0.0015 increase mean depth and steepness characteristics - at least out to the 30-foot depths. There perhaps may be a slight indication that waves with steepness ratios of about 0.01 decrease the profile steepness, but reliable data are not available in sufficient quantity.

If, in general, more information is desired it probably will be necessary to consider a beach which is attacked by waves from only one direction. At the same time the steepness ratio of the waves should vary more than in the case considered above. Profiles in the middle of a long bay probably would be the best. Then the statistical treatment could be divided into considerations based partly on waves with the same length and partly on waves with the same steepness ratio and the same wave effect. This naturally will be difficult. Another way of attacking the problem possibly is by investigating which steepness ratio for a given wave length (or a given wave effect), a given depth, and a given material will give the most predominant drift in the direction of wave propagation. This, among other things, would include a study of the ripple marks. At the same time the maximum depth of bed load transportation by oscillating wave motion alone should be investigated and finally the effect of bottom slope should also be included.

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APPENDIX A - THE THYBOROEN PROJECT

The coastal protection works carried out to date have limited only the nearshore erosion. At the same time the beach profiles have steepened because the groins to a certain degree maintained the shore line.(see section 3.12, Table 2), while they did not influence the erosion of the bottom outside the groins (section 3.25 Table 13). As mentioned in section 3.24 the maximum steepness occurred in 1934 and the shore line recession increased since about 1934 (see section 3.12, Tables 2 and 3). The groins still play a role in decreasing the nearshore littoral drift, but because of continuous erosion they are no longer maintained to their full lengths, in many cases outer sections, 200 to 400 feet long, have been abandoned. (see section 2.3).

As the measures against erosion to date have been unsatisfactory, new provisions are under consideration. The latest measures are authorized by the Act of August 1946, which provided for the construction of two large jetties, one on either side of the channel, and a new solid dike about $1\frac{1}{2}$ miles from the sea. The dike is to be built across the channel as a dam with sluices, as shown in Figure A-1, where the location of the dike is indicated by dotted lines.

This project is very expensive (it would cost about 60 million dollars if carried out in the United States) and the entire design is not too good from a technical point of view. The foundation conditions -- a very soft clay (shear 600-800 pounds per square foot) -- are bad, but even worse is the fact that the project, from a coastal engineering point of view, will not work well.

The quantity of sand which deposits annually in the shallows of the inlet, according to Table 1, is about one million cubic yards. This corresponds, within the degree of accuracy of measurement, with the quantity eroded from the barriers between +13 and -20 feet under the assumption of an average shore line recession of about 10 feet. (As mentioned in section 1 the subsoil is composed of clay below -20 feet). Since the most recent soundings in 1936-1938 and 1950 show that erosion takes place out to depths of about 65 feet or more, it is not unreasonable to assume that the average annual shore line recession on the Barriers after closing the channel will be

$$10 \text{ feet} \times \frac{65 - 20}{65 + 13} = \text{about } 6 \text{ feet}$$

However, with the shape of the northern jetty as indicated on Figure A-1, some sand will probably be carried in between the jetties where it will settle and the shore line recession, therefore, must be expected to be somewhat greater, perhaps about 7 feet. As far as can be seen from older maps (see section 2.21, etc.), the shore line recession on the unbroken barrier was about 10 feet a year, but at that time all the material eroded

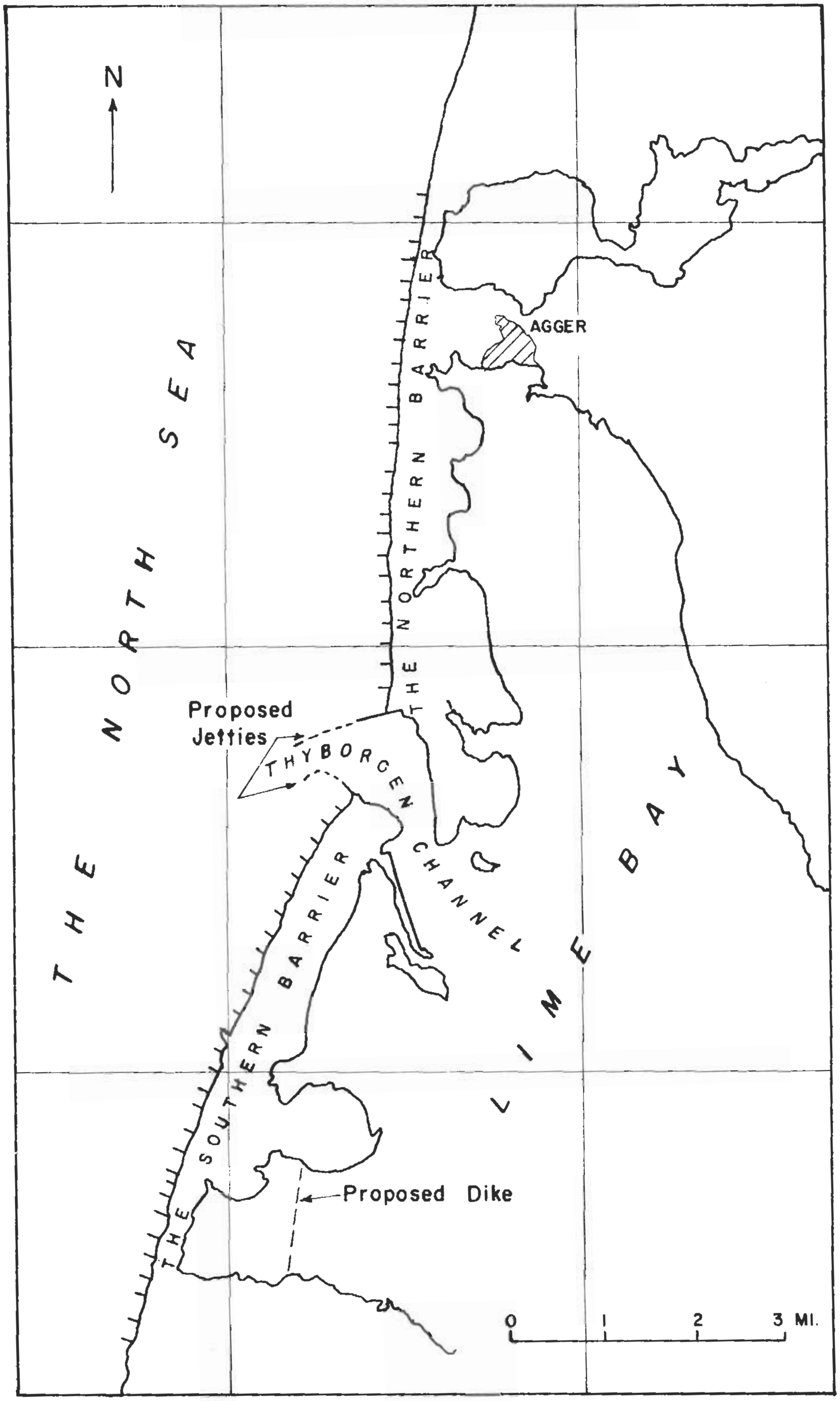

FIGURE A-1 THE THYBOROEN PROJECT

from the Barriers drifted away from the area, while after closing the channel most of the sand eroded will probably remain in the area outside the Barrier until the shore line is again more nearly straight. This, according to the above, corresponds to a decrease in shore line recession of only 3 to 4 feet. It appears, therefore, that the new expensive project considered as a coastal protection project cannot be expected to answer its purpose too well.

ERRATA SHEET

for

Technical Memorandum No. 44 of the Beach Erosion Board
"Coast Erosion and the Development of Beach Profiles"

Page 19 - 3rd line from bottom of page - change "are very small" to read "and very small"

Page 23 - 5th line of paragraph 3.23 - change "and 20 and 230 foot depth contours" to read "and 20 and 30-foot depth contours"

Page 52 - Figure 21 - both horizontal and vertical distances are given in meters

Page 53 - 9th line from top of page - change "partially" to read "particularly"

Page 55 - 2nd line of 4th paragraph - make "dx" read "dx_1"

Page 57 - Table 29, 1st item under 1st column headed "Periods" - "1921-1924" should read "1921-1934"

Page 60 - 8th line of paragraph 3.76 - insert "and" after Northern Barrier.

Page 71 - Figure 24 - change equation for summer profile, "$y^{3/2} = 0.01x$" to read "$y^{3/2} = 0.1x$"

Coordinates in feet of plotted points are:

Summer y	Summer x	Winter y	Winter x
0	0	0	0
10	360.10	10	450.10
20	900.20	20	945.20
30	1680.30	30	1710.30

Approximate scale for Figure 24 is 1" = 40' vertical and 1" = 300' horizontal.

Page 72 - Figure 25, Title - change "BOVDJAERG" to read "BOVBJAERG"

Page A-2 - Figure A-1 - substitute the attached Figure A-1 which shows the location of the proposed dike in entirety.

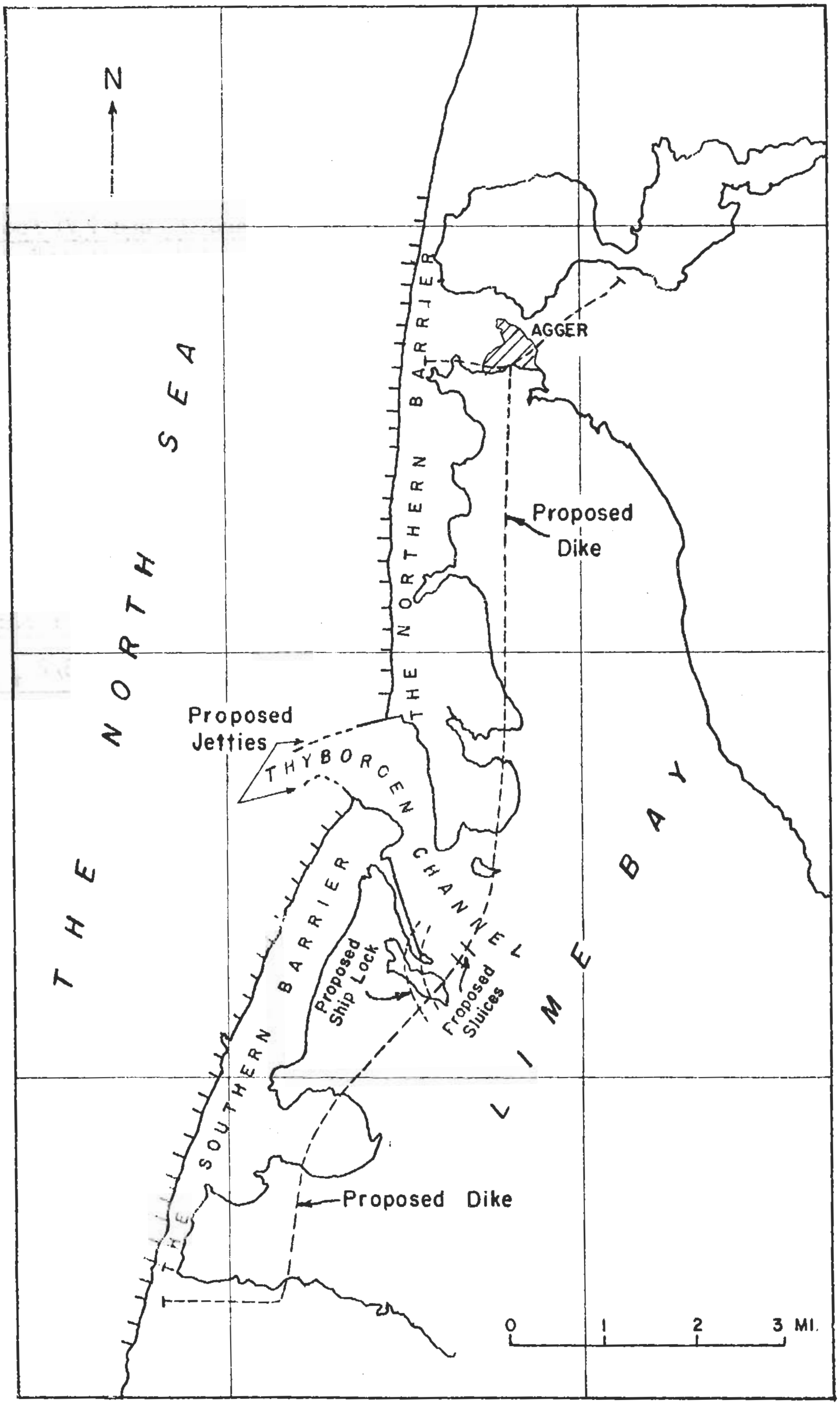

FIGURE A-1 THE THYBOROEN PROJECT