CONNECTICUT RIVER FLOOD CONTROL PROJECT

HARTFORD, CONN.

CONNECTICUT RIVER CONNECTICUT

ANALYSIS OF DESIGN FOR PARK RIVER PRESSURE CONDUIT

DOUBLE-BARREL CONDUIT METHODS OF COMPUTATION

ITEM H1.6 CONTRACT

1941

CORPS OF ENGINEERS, U. S. ARMY

U. S. ENGINEER OFFICE PROVIDENCE, R. I.

CONNECTICUT RIVER FLOOD CONTROL

ANALYSIS OF DESIGN

PARK RIVER PRESSURE CONDUIT

HARTFORD, CONNECTICUT

CORPS OF ENGINEERS, UNITED STATES ARMY

UNITED STATES ENGINEER OFFICE PROVIDENCE, RHODE ISLAND

PARK RIVER PRESSURE CONDUIT
ITEM Ht.6

1. General description. - The Park River pressure conduit is to be a double box of reinforced concrete with center wall. It has been designed as a hingeless arch, with the weight of the structure and superimposed loads assumed to be uniformly distributed over the foundation where the structure is founded on earth or on piles. Where the structure rests on rock, only the dead weight of the structure is assumed to be uniformly distributed over the foundation; the superimposed loading being assumed to be concentrated at the bases of the vertical walls.

2. Method of analysis. - In analyzing the conduit, the center wall is at first assumed to be out. The analysis is made in accordance with standard arch formulae as given in Spofford's "Theory of Structures," page 539, the symbols used by Metcalf & Eddy corresponding to those used in Spofford's. Bending moments were computed for one half the conduit, with center wall out, for all symmetrical loading, the bending moments for the other half being the same by symmetry. With the bending moments known, the deflection at the center of the conduit, with the center wall out, was then figured from the general formula $y = \int Mxdx/E.I.$ modified in this case to the form $\Delta y = \frac{1}{E} \sum \frac{Mx}{t^3}$
The deflection caused by a load of unity at the center of the conduit with center wall out was then figured. The ratio of the deflection for dead and superimposed loads to the deflection caused by a load of unity gives the vertical reaction taken by the center wall.

With the center wall reaction known, bending moments were again figured on the entire conduit section caused by this reaction only. The

algebraic sum of the bending moments from dead load and superimposed load with center wall out, plus the moments caused by the center wall reaction, gives final bending moments on the conduit.

3. Unsymmetrical loading. - For unsymmetrical loading the superimposed load is assumed to cover one-half the conduit roof and the thrust against the side wall of the loaded half is assumed to be greater than the thrust against the side wall of the unloaded half. The result of the unsymmetrical loading is to produce rotation and translation at the top of the center wall, and to produce bending moments in the center wall, roof slab, and floor slab.

To solve the bending moments caused by center wall movements noted above, use was made of the theory of slope deflections. The general formulae for slope deflections are given in Ketchum's "Steel Mill Building," page 298. The angle of rotation is given by the general formula $\Delta\theta = \int \frac{Md\varsigma}{EI}$

Horizontal displacement, or translation, is given by the general formula $\Delta X = \int \frac{Myd\varsigma}{EI}$

These formulae have been modified into the forms $\Delta X = \frac{\Delta\varsigma}{E} \sum \frac{My}{t^3}$ and $\Delta\theta = \frac{\Delta\varsigma}{E} \sum M/t^3$ By means of the slope deflection formulae, two simultaneous equations are established which have two unknowns, these being the bending moments at the top and bottom of the center wall due to the movement of the top of the wall. The bending moments at the top and bottom of the center wall due to movement are added to the center line moments in the roof and floor slabs due to dead load and superimposed load to obtain final moments.

The final section was determined by the maximum conditions as obtained by making combinations of the results of the various assumed loadings.

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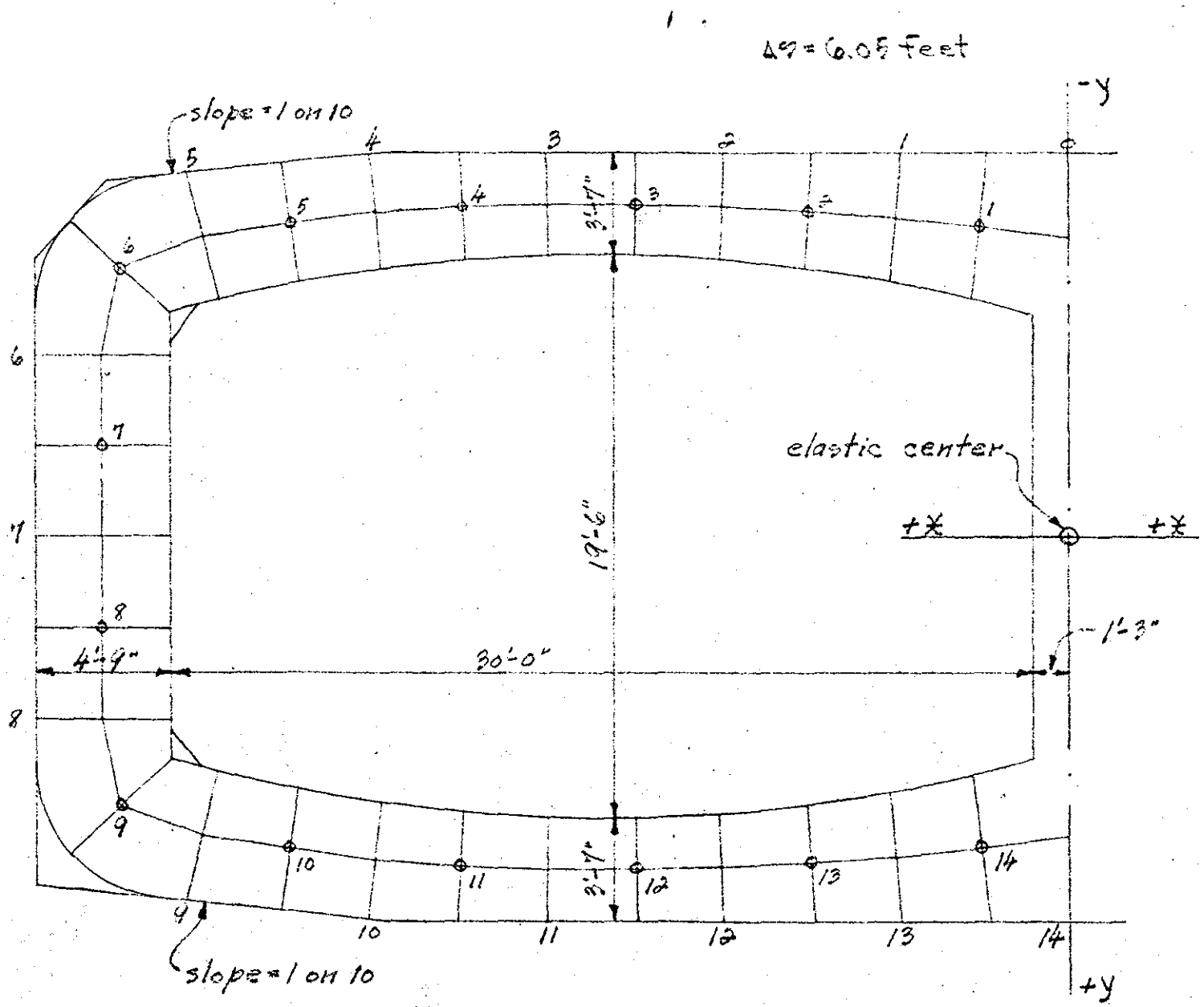
Subject: Design of a double barrel conduit

Computation: Half section of conduit

Computed by: W.C.O. Checked by: Date:

Page 1



Half Section of Conduit

Scale: 3/16" = 1'-0"

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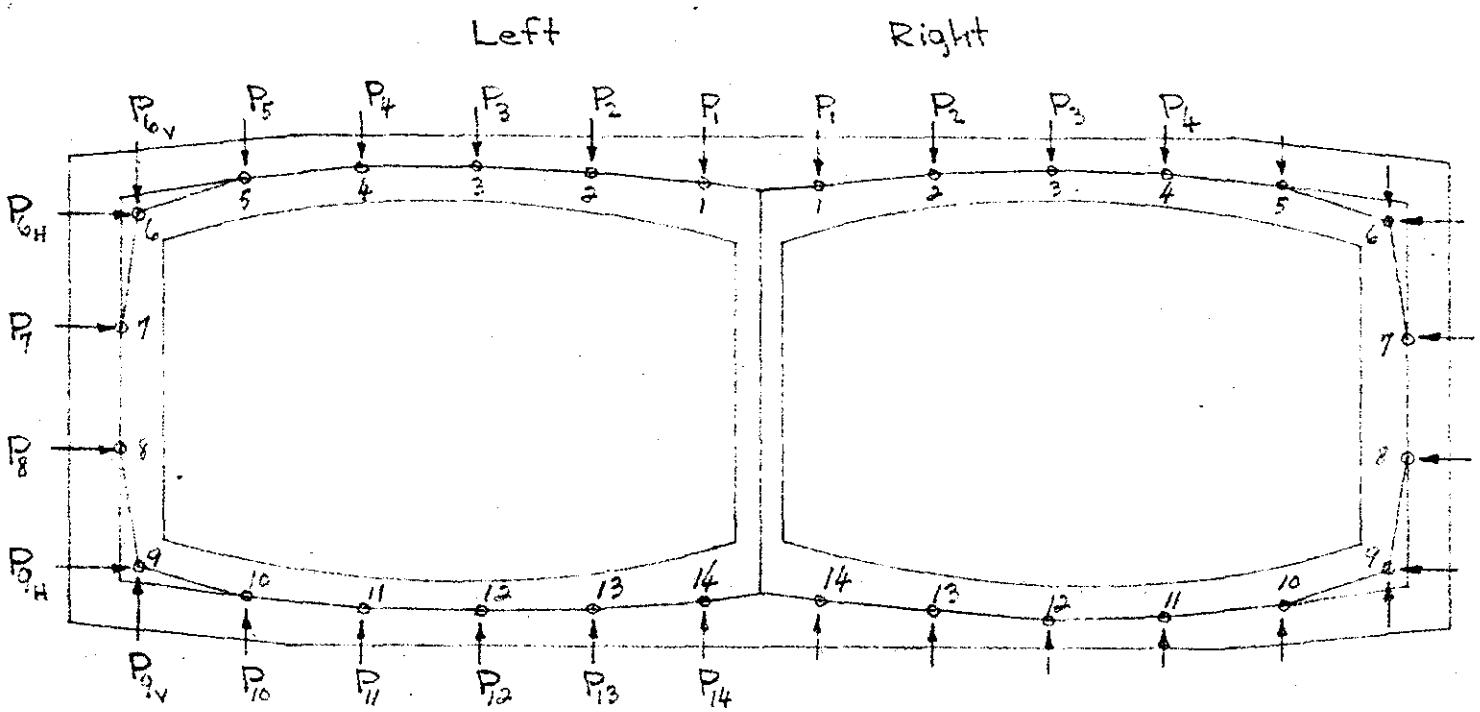

For cases involving symmetrical loadings, investigate one half of the section only.

For cases involving unsymmetrical loadings, investigate the whole section by combining the effects of loading on the left half with the effects of loading on the right half.

Subject: Design of a double barrel conduit
Computation: Conditions of loading
Computed by: W.C.O. Checked by: Date

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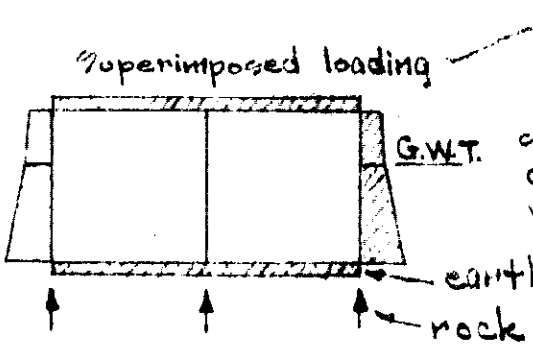
The following cases to be combined for maximum moments, thrusts and shears.

Dead wt. of structure only

Case I

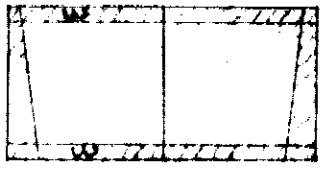
Wt. of structure to be distributed uniformly on earth or rock.

Case II



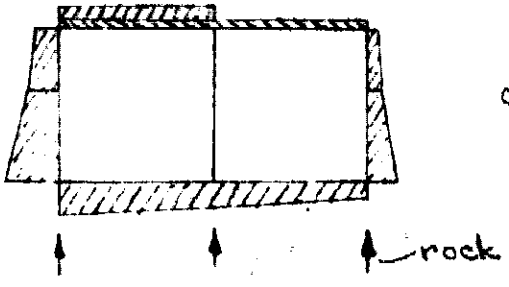
When structure rests on rock, superimposed load to be considered as concentrated at bases of vertical walls

Case III



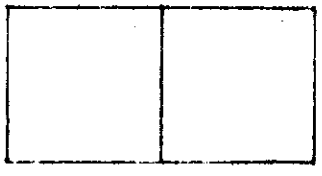
w = pressure per sq. foot from flood waters at elevation 46.0

Case IV



Unsymmetrical loading; stresses computed with dead wt. of structure.

Case V



Shrinkage

Assume shrinkage equal to a drop in temperature of 35°

Subject: Design of a double barrel conduit
Computation: Symbols
Computed by: W. C. O. Checked by: Date

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Symbols

x = distance left or right of ℄ ; plus sign either direction.

y = distance above or below elastic center; minus above, plus below.

z = distance to elastic center from intersection of center lines of top slab and middle wall.

t = total thickness of concrete section.

V' = total shear from vertical loads at any section for a simple cantilever beam.

H' = total shear from horizontal loads at any section for a simple cantilever beam.

M'_V = total moment from vertical loads at any section for a simple cantilever beam.

M'_H = total moment from horizontal loads at any section for a simple cantilever beam.

$M' = \Sigma (M'_V + M'_H)$

m' = total moment at any section from a unit load applied at center of middle wall and acting in same direction as reaction of middle wall.

M_0 = computed moment at elastic center from exterior loads.

m_0 = " " " " " " unit load.

H_0 = " thrust " " " " exterior loads.

h_0 = " " " " " " unit load.

M_a = moment at any section of conduit without middle wall.

M_c = " " " " " " with middle wall acting and unsymmetrical loading.

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Symbols (cont'd)

M = final corrected moment at any section.

N = thrust acting perpendicular to section; plus = tension, minus = compression.

V = shear at any section of conduit; plus = shear acting outward, minus = shear acting inward.

V_0 = computed shear at elastic center from external loads.

bject .. Design of a double barrel conduit
omputation Equations
omputed by W.C.O. Checked by Date

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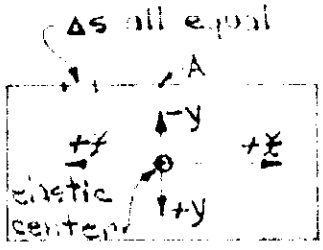

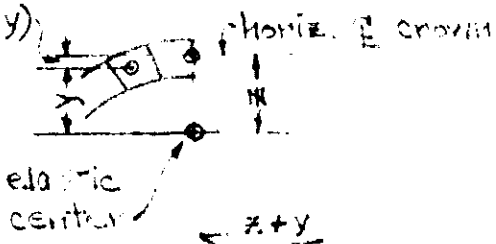

$$z = \frac{\sum \frac{z+y}{t^3}}{\sum \frac{1}{t^3}}$$

Hingeless Arch Equations (see J.R.E.T. vol. 2)

$\int \frac{y}{I} = 0$ and $ds = \Delta s$ = a constant

$$H_o = -\frac{\sum \frac{M'y}{t^3}}{\sum y^2/t^3} \qquad M = M' + M_o + H_o y + V_o x$$

$$V_o = \frac{\sum \frac{M'x}{t^3} - \sum \frac{M'_R x}{t^3}}{\sum x^2/t^3}$$

$$M_o = -\frac{\sum M'/t^3}{\sum 1/t^3}$$

Vertical displacement at center wall

m = moment at any section from unit load acting up at "A".

Let (½R) = Load on center wall from one half of structure.

$\Delta y = \frac{1}{E} \sum \frac{Mx}{t^3}$ deflection at "A" without center wall.

$\Delta y' = \frac{1}{E} \sum \frac{\frac{1}{2}Rmx}{t^3}$ " " " due to force of ½R acting at "A".

neglect shortening of wall, then

$\Delta y = \Delta y'$

$$\tfrac{1}{2}R \sum \frac{mx}{t^3} = -\sum \frac{Mx}{t^3}$$

$$\tfrac{1}{2}R = -\frac{\sum \frac{Mx}{t^3}}{\sum \frac{mx}{t^3}}$$

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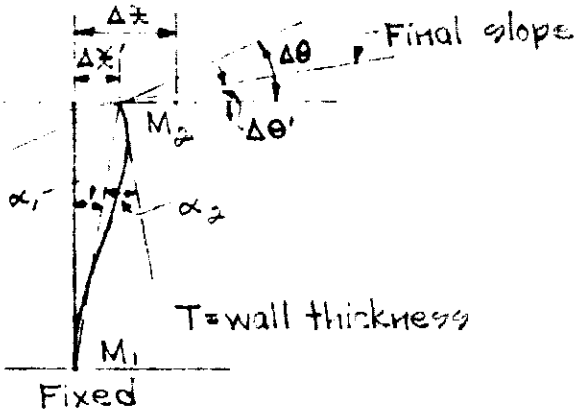
Computation Equations
Computed by W.C.O. Checked by Date

Moments at top and bottom of center wall due to movement at top of center wall when conduit is unsymmetrically loaded.



Rotation

$$M_1 = \frac{2ET^3}{h}(2\alpha_1 + \alpha_2)$$

$$M_2 = \frac{2ET^3}{h}(2\alpha_2 + \alpha_1)$$

$$\alpha_2 - \alpha_1 = \frac{h}{2ET^3}(M_2 - M_1) = \Delta\theta'$$

$\Delta\theta - \Delta\theta'$ = Angle of rotation

or, $\Delta\theta - \frac{h}{2ET^3}(M_2 - M_1)$ = Angle of rotation

$$\Delta\theta = \frac{\Delta s}{E}\sum\frac{M}{t^3}$$

Angle of rotation $= \frac{M_2}{2}\cdot\frac{\Delta s}{E}\sum\frac{m}{t^3}$ { m = moment at any section due to unit moment on ½ frame acting at elastic center.

$$\frac{\Delta s}{E}\sum\frac{M}{t^3} - \frac{h}{2ET^3}(M_2 - M_1) = -\frac{\Delta s}{E}\cdot M_2\cdot\tfrac{1}{2}\sum\frac{m}{t^3}$$

E.C.-4, m=1

$$\boxed{\sum\frac{M}{t^3} - \frac{h}{\Delta s}\cdot\frac{(M_2 - M_1)}{2T^3} = -\tfrac{1}{2}M_2\sum\frac{m}{t^3}}$$ (Final equation)

M = final moment at every section

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Computation Equations
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Horizontal displacement

$$\Delta x' = \frac{h^2}{6ET^3}(2M_1 - M_2)$$

$\Delta x - \Delta x'$ = Horizontal displacement

or, $\Delta x - \frac{h^2}{6ET^3}(2M_1 - M_2) =$ " "

$$\Delta x = \frac{\Delta s}{E} \sum \frac{My}{t^3}$$

Horiz. displacement $= H \cdot \frac{\Delta s}{E} \cdot \frac{1}{2} \sum \frac{my}{t^3}$ { m = moment at any section due to unit horiz. load acting at elastic center. Sum for ½ of frame.

$$\frac{\Delta s}{E} \sum \frac{My}{t^3} - \frac{h^2}{6ET^3}(2M_1 - M_2) = -H \cdot \frac{\Delta s}{E} \cdot \frac{1}{2} \sum \frac{my}{t^3}$$

{ $m = 1 \times y = y$
$H = \frac{M_1 + M_2}{h}$

$$\boxed{\sum \frac{My}{t^3} - \frac{h^2}{\Delta s} \cdot \frac{(2M_1 - M_2)}{6T^3} = -\frac{M_1 + M_2}{h} \cdot \frac{1}{2} \cdot \sum \frac{y^2}{t^3}}$$ (Final equation)

Summary

1.) $\sum \frac{M}{t^3} - \frac{h}{2\Delta s T^3}(M_2 - M_1) = -\frac{1}{2} M_2 \sum \frac{m}{t^3}$

2.) $\sum \frac{My}{t^3} - \frac{h^2}{\Delta s} \cdot \frac{(2M_1 - M_2)}{6T^3} = -\frac{M_1 + M_2}{h} \cdot \frac{1}{2} \cdot \sum \frac{y^2}{t^3}$

3.) $H = \frac{M_1 + M_2}{h}$

3 unknowns and 3 equations

solve for M_1, M_2, and H

Subject Design of a double barrel conduit

Computation Equations

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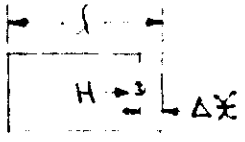
Temperature change



shrinkage of top slab only = Δx

apply force "H" to elastic center so that $\Delta x = 0$

$\Delta x = cTl$, $c = .0000065$, T = degrees of temp. , l = length in ft.

$H \cdot \Delta s \cdot \Sigma \frac{my}{EI} = \Delta x$ m = unit load acting same as "H", m = y

E = 144 × 2,500,000 = 360,000,000 ft. Units

$H \cdot \Delta s \cdot 12 \Sigma \frac{y^2}{Et^3} = ctl$ $I = t^3/12$, t in ft.

$$\boxed{H = \pm \frac{cTlE}{12 \Delta s \Sigma \frac{y^2}{t^3}}}$$ (minus for shrinkage)

c = coef. of expansion.

T = assumed change of temp. in degrees.

l = length of one half of section in feet.

E = modulus of elasticity in foot units.

Δs = length of one division of axis in feet.

Subject: Design of a double barrel conduit
Computation: Frame constants
Computed by: W.C.D. Checked by: Date:

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Frame constants

$y = (z+y) - z$

Point	t	$\frac{1}{t^3}$	$z+y$	$\frac{z+y}{t^3}$	y	$\frac{y}{t^3}$	$\frac{y^2}{t^3}$	x	$\frac{x}{t^3}$	$\frac{x^2}{t^3}$
	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
etc.										

$$z = \frac{\sum \frac{z+y}{t^3}}{\sum \frac{1}{t^3}} = \frac{\sum \text{col. } 4}{\sum \text{col. } 2}$$

Solution of "m_0" and "h_0"

$m = m' + m_0 + h_0 y$

Point	Load	$m' = x$	$\frac{1}{t^3}$	$\frac{m'}{t^3} = \frac{x}{t^3}$	y	$\frac{m'y}{t^3}$	$h_0 y$	m	$\frac{m\,x}{t^3}$
	11	12	13	14	15	16	17	18	19
1	unit load @ center wall								
2									
3									
etc.									

$$m_0 = -\frac{\sum \frac{m'}{t^3}}{\sum \frac{1}{t^3}} = -\frac{\sum \text{col. } 14}{\sum \text{col. } 13} \quad ; \quad h_0 = -\frac{\sum \frac{m'y}{t^3}}{\sum \frac{y^2}{t^3}} = -\frac{\sum \text{col. } 16}{\sum \text{col. } 7}$$

should be equal to zero for section symm. about a horiz. axis

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Subject: Design of a double barrel conduit

Computation: Symmetrical loading

Computed by: W. C. O. Checked by: Date:

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Signs:	Vert. loads	Horiz. loads	Δx	Δy
	Down = minus Up = plus	Inward = minus Outward = plus	Top slab = plus Bot. slab = minus	Down = plus Up = minus

	Hor. proj.	Vert. Loads	Shear V'	Δx	$\Delta M'_V = V'\Delta x$	M'_V	Hor. Forces	Shear H'	Δy	$\Delta M'_H = H'\Delta y$	M'_H	$M' = M'_V + M'_H$
	20	21	22	23	24	25	26	27	28	29	30	31
		P_1	P_1	(Δx_{1-2})	0	0	0	0				
		P_2	P_1+P_2	(Δx_{2-3})	$P_1 \cdot \Delta x_{1-2}$	$0+\Delta M'_{V_2}$	0	0				
		P_3	$P_1+P_2+P_3$	(Δx_{3-4})	etc.	$M'_{V_2}+\Delta M'_{V_3}$	0	0				
							P_6	P_6	Δy_{6-7}	0	0	
			etc.	etc.		etc.	P_7	P_6+P_7	Δy_{7-8}	$P_6 \cdot \Delta y_{6-7}$	$0+\Delta M'_{H_7}$	
							P_8	$P_6+P_7+P_8$	Δy_{8-9}	etc.	$M'_{H_7}+\Delta M'_{H_8}$	
		P_{13}					etc.	etc.	etc.		etc.	
		P_{14}	0				0					

$M_\square = M' + M_o + H_o y$

M'	$\frac{1}{t^3}$	$\frac{M'}{t^3}$	y	$\frac{M'y}{t^3}$	$H_o y$	$M_\square$	$\frac{x}{t^3}$	$\frac{M_\square x}{t^3}$	m	$\frac{1}{2}Rm$	Final M	N	V
31	13	32	15	33	34	35	14	36	18	37	38	39	40

$$M_o = -\frac{\sum \frac{M'}{t^3}}{\sum \frac{1}{t^3}} = -\frac{\sum \text{col. } 32}{\sum \text{col. } 13}$$

$$H_o = -\frac{\sum \frac{M'y}{t^3}}{\sum \frac{y^2}{t^3}} = -\frac{\sum \text{col. } 33}{\sum \text{col. } 7}$$

$$\tfrac{1}{2}R = -\frac{\sum \frac{M_\square x}{t^3}}{\sum \frac{mx}{t^3}} = -\frac{\sum \text{col. } 36}{\sum \text{col. } 19}$$

Find "N" and "V" either graphically or by trigonometry.

℄ moments

solve for $M_{\text{℄}}$: same method for $M_{\text{℄}}$ at bottom.

For signs, see symmetrical loading.

$M' = M'_V + M'_H$

	Point	Vert. Loads	V'	Δx	$\Delta M'_V = V'\Delta x$	M'_V	Hor. Loads	H'	Δy	$\Delta M'_H = H'\Delta y$	M'_H	M'	$\frac{1}{t^3}$	$\frac{M'}{t^3}$	$\frac{y}{t^3}$	$\frac{M'y}{t^3}$	$\frac{x}{t^3}$	$\frac{M'x}{t^3}$
	0	41	42	43	44	45	46	47	48	49	50	51	13	52	6	53	9	54
Left	1	P_1	P_1		0	0	0	0		0	0							
	2	P_2	P_1+P_2	Δx_{1-2} Δx_{2-3}	$P_1\Delta x_{1-2}$	$0+P_1\Delta x_{1-2}$	0	0		0	0							
							P_6	P_6	Δy_{6-7}	0	0							
			etc.	etc.	etc.	etc.	P_7	P_6+P_7	Δy_{7-8}	$P_6\Delta y_{6-7}$	$0+P_6\Delta y_{6-7}$							
	13	P_{13}					etc.	etc.	etc.	etc.	etc.							
	14	P_{14}																
Right	1																	
	2																	
				same procedure as above														
	13																	
	14																	

$$M_0 = -\frac{\sum \frac{M'}{t^3}}{2\sum \frac{1}{t^3}} = -\frac{\sum \text{col. } 52}{2\sum \text{col. } 13}$$

$$H_0 = -\frac{\sum \frac{M'y}{t^3}}{2\sum \frac{y^2}{t^3}} = -\frac{\sum \text{col. } 53}{2\sum \text{col. } 7}$$

$$V_0 = \frac{\sum \frac{M'_L x}{t^3} - \sum \frac{M'_R x}{t^3}}{2\sum \frac{x^2}{t^3}} = \frac{\sum \text{col. } 54 \text{ (left)} - \sum \text{col. } 54 \text{ (right)}}{2\sum \text{col. } 10}$$

Subject: Design of a double barrel conduit

Computation: Unsymmetrical loading (cont'd)

Computed by: W. E. O. Checked by: Date:

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$M_a = M' + M_o + H_o y + V_o x$

$M_c = M_a + \frac{1}{2} R m$

	Point	y	$H_o y$	x	$V_o x$	M'	M_a	$\frac{x}{t^3}$	$\frac{M_a x}{t^3}$	m	$\frac{1}{2} R m$	M_c
	0	5	55	8	56	51	57	9	58	18	59	60
Left	1											
	2											
	3											
	4											
	5											
	13											
	14											
Right	1											
	2											
	3											
	4											
	5											
	13											
	14											

check: $\Sigma \frac{M_{a_L} x}{t^3} = \Sigma \frac{M_{a_R} x}{t^3}$ or, Σ col. 58 (left) = Σ col. 58 (right)

$$\frac{1}{2} R = -\frac{\Sigma \frac{M_a x}{t^3}}{2 \Sigma \frac{m x}{t^3}} = -\frac{\Sigma \text{ col. } 58}{2 \Sigma \text{ col. } 19}$$

Subject: Design of a double barrel conduit
Computation: Unsymmetrical loading (cont'd)
Computed by: W.C.O. Checked by: Date:

$$\tfrac{1}{2}\sum\frac{M_c}{t^3}-\frac{h}{\Delta s}\cdot\frac{(M_2-M_1)}{2T^3}=-\tfrac{1}{2}M_2\sum\frac{1}{t^3}$$

$$\tfrac{1}{2}\sum\frac{M_c y}{t^3}-\frac{h^2}{\Delta s}\cdot\frac{(2M_1-M_2)}{6T^3}=-\frac{M_1+M_2}{h}\cdot\tfrac{1}{2}\sum\frac{y^2}{t^3}$$

$$H=\frac{M_1+M_2}{h}$$

solve above equations for M_1, M_2, and H; then substitute necessary quantities in table

$$M=M_c+\tfrac{1}{2}H(z+y)+\frac{M_2}{2}$$

	Point	M_c	$\frac{1}{t^3}$	$\frac{M_c}{t^3}$	$\frac{y}{t^3}$	$\frac{M_c y}{t^3}$	$z+y$	$\frac{H}{2}(z+y)$	$\frac{M_2}{2}$	Final M	N	V
		60	13	61	6	62	3	63	64	65	66	67
Left	1											
	2											
	13											
	14											
Right	1							same as above except opposite signs.	opposite sign to above			
	2											
	13											
	14											

Total crown thrust $= H_o \pm \frac{H}{2}$

Note: "H" either acts with or against H_o and since $\tfrac{1}{2}H$ is assumed to be taken by each half of the section, the thrust on one will be $H_o+\tfrac{1}{2}H$ and on the other half will be $H_o-\tfrac{1}{2}H$.

Checks

$$\sum\frac{M_c}{t^3}(\text{left})=-\sum\frac{M_c}{t^3}(\text{right})$$

$$\sum\frac{M_c y}{t^3}(\text{left})=\sum\frac{M_c y}{t^3}(\text{right})$$

℄ moments

use method used for symmetrical loading: in this case consider also the moments at the ℄ due to movement of the center wall.

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bject Design of a double barrel conduit
Computation Temperature
Computed by W.C.O. Checked by Date

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$H_o y$ $M = M_\square + \frac{1}{2} R m$

Point	y	$M_\square$	$\frac{x}{t^3}$	$\frac{M_\square x}{t^3}$	m	$\frac{1}{2}Rm$	Final M	N	V
	5	68	9	69	18	70	71	72	73
1									
2									
⋮									
12									
13									
14									

$$H_o = \pm \frac{cT\ell E}{12 \Delta s \sum y^2/t^3}$$

$$\frac{1}{2}R = -\frac{\sum \frac{M_\square x}{t^3}}{\sum \frac{m x}{t^3}} = -\frac{\sum \text{col. } 69}{\sum \text{col. } 19}$$ {usually small enough to be neglected.